ECE544: Fault-Tolerant Computing & Reliability Engineering
(Fall 2022)

Homework #2 Solution
(45 points)

1. How many check bits are needed if the Hamming correcting code is used to detect single bit errors in a 64-bit data word? (**5 points**)

Solution:

Need K check bits such that: $64 + K \leq 2^K - 1$.
The minimum value of K, which satisfies this condition, is 7.

2. Develop an SEC code for a 16-bit data word. 1) Generate the code for the data word *0101000000111001*. 2) Show that the code will correctly identify an error in data bit D_{16}. (**40 points**)

Solution:

1) (30 points)

Step 1 (4 points): According to the inequality 2^k-1>=M+K, where M=16, 5 check bits are needed for an SEC code for 16-bit data words.

Step 2 (9 points): The layout of data bits and check bits:

Bit Position	Position Number	Check Bits	Data Bits
21	10101		D_{16}
20	10100		D_{15}
19	10011		D_{14}
18	10010		D_{13}
17	10001		D_{12}
16	10000	C16	
15	01111		D_{11}
14	01110		D_{10}
13	01101		D_9
12	01100		D_8
11	01011		D_7
10	01010		D_6
9	01001		D_5
8	01000	C8	
7	00111		D_4
6	00110		D_3

5	00101		D_2
4	00100	C4	
3	00011		D_1
2	00010	C2	
1	00001	C1	

Step 3 (15 points): The check bits are calculated

$C1 = D1 \oplus D2 \oplus D4 \oplus D5 \oplus D7 \oplus D9 \oplus D11 \oplus D12 \oplus D14 \oplus D16$

$C2 = D1 \oplus D3 \oplus D4 \oplus D6 \oplus D7 \oplus D10 \oplus D11 \oplus D13 \oplus D14$

$C4 = D2 \oplus D3 \oplus D4 \oplus D8 \oplus D9 \oplus D10 \oplus D11 \oplus D15 \oplus D16$

$C8 = D5 \oplus D6 \oplus D7 \oplus D8 \oplus D9 \oplus D10 \oplus D11$

$C16 = D12 \oplus D13 \oplus D14 \oplus D15 \oplus D16$

For the word $D_{16}D_{15}D_{14}\ldots D_2D_1$=**0101000000111001**, the check bits are
C16 = 0; C8 = 0; C 4 = 0; C2 = 0; C1 = 1.

Step 4 (2 points): The code word is **010100000001101000101**

2) (10 points)

If an error occurs in data bit D_{16}, the check bits become
C16 = 1; C8 =0; C4 = 1; C2 = 0; C1 = 0.

Comparing the two sets of check bits forms the syndrome word:

	C16	C8	C4	C2	C1
	0	0	0	0	1
$\oplus$	1	0	1	0	0
	1	0	1	0	1

The result indicates an error identified in bit position 21, which is data bit D_{16}