

A verification example

(1 0 1 0 1 0 1)

$$M = 4$$

$$D_4 D_3 D_2 D_1 = 1001$$

① step 1: To determine how many check bits are needed for correcting any single-bit errors:

$$2^k - 1 \geq M + k \quad \text{where } M = 4$$

$$\left. \begin{array}{l} 2^2 - 1 < 4 + 2 \\ 2^3 - 1 = 4 + 3 \end{array} \right\} \Rightarrow k = 3$$

step 2: Arrange 4-bit data word and ~~3~~ check bits into 7-bit code word. s.t. the bit positions whose position # are power of 2 are assigned to check bits

Bit pos.	7	6	5	④	3	②	①
pos. #	111	110	101	100	011	010	001
check bit				C_4		C_2	C_1
Data bit	D_4	D_3	D_2		D_1		

step 3: Using the check-bit generating rule, we obtain
(by exclusive-oring all data bits whose pos. # has "1" at the same bit as the pos. # of the check bit)

$$C_1 = D_1 \oplus D_2 \oplus D_4 = 1 \oplus 0 \oplus 1 = 0$$

$$C_2 = D_1 \oplus D_3 \oplus D_4 = 1 \oplus 0 \oplus 1 = 0$$

$$C_4 = D_2 \oplus D_3 \oplus D_4 = 0 \oplus 0 \oplus 1 = 1$$

So, the code word for 1001 will be "100|100" (stored in mem)

Error-detection capability:

(2) Assume an error occurs in D_4 , the data word read out from memory becomes $D_4 D_3 D_2 D_1 = 0001$.
Regenerate the check bit (repeat step 3), we got a new set of check bits:

$$C_1' = 1 \oplus 0 \oplus 0 = 1$$

$$C_2' = 1 \oplus 0 \oplus 0 = 1$$

$$C_4' = 0 \oplus 0 \oplus 0 = 0$$

The syndrome word is the exclusive-or of two sets of check bits.

	C_4	C_2	C_1
old:	1	0	0
\oplus new	0	1	1
	1	1	1

Observations:

(a) The syndrome word has more than one 1s, an error has occurred in some data bit

(b) The value of the syndrome word indicates the position of the data bit in error, that is, position #7, D_4 .

We can correct the error by simply converting D_4 from $0 \rightarrow 1$.

(3) Assume an error occurs in C_1 , the data word read out from memy is 1001, when regenerating check bits, we got $C_1' C_2' C_4' = 001$. The check bits read from memory is $C_1 C_2 C_4 = 101$
Syndrome word:

	C_4	C_2	C_1
old	1	0	1
\oplus New	1	0	0
	0	0	1

observation:
The syndrome word contains only one 1, an error has occurred in the check bit!