

Review Question

- An on-line computer system has five incoming communication lines with the properties described in the following table.

Line	Fraction of traffic	Fraction of message without error
1	0.1	0.97
2	0.1	0.96
3	0.4	0.99
4	0.1	0.97
5	0.3	0.98

- What is the probability that a randomly chosen message has been received without error?
- Suppose a message selected at random is found to be free of error. What is the probability that this message was from communication line 3?

Solution

Define events:

A: a chosen msg is received w/o error

B_i : a chosen msg is from comm. line # i , $i=1,2,3,4,5$

$$P(B_1) = 0.1 \quad P(B_2) = 0.1 \quad P(B_3) = 0.4 \quad P(B_4) = 0.1 \quad P(B_5) = 0.3$$

$$P(A|B_1) = 0.97 \quad P(A|B_2) = 0.96 \quad P(A|B_3) = 0.99 \quad P(A|B_4) = 0.97 \quad P(A|B_5) = 0.98$$

a) According to "Total Prob Theorem", we have

$$\begin{aligned} P(A) &= \sum_{i=1}^5 P(A|B_i) \cdot P(B_i) = 0.1 \times 0.97 + 0.1 \times 0.96 + 0.4 \times 0.99 + 0.1 \times 0.97 \\ &\quad + 0.3 \times 0.98 \\ &= 0.98 \end{aligned}$$

$$\begin{aligned} \text{b) } P(B_3|A) &= \frac{P(A|B_3) \cdot P(B_3)}{P(A)} \\ &= \frac{0.99 \times 0.4}{0.98} = 0.404081633 \end{aligned}$$