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Solution to problem

		First die					
Sum		1	2	3	4	5	6
second die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$$P(A) = P(\text{1st die results in a } 1/2/3) = \frac{1}{2}$$

$$P(B) = P(\text{2nd die results in a } 4/5/6) = \frac{1}{2}$$

$$P(C) = P(\text{the sum of the two faces is } 7) = \frac{6}{36} = \frac{1}{6}$$

$$A \cap B = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\} \Rightarrow P(A \cap B) = \frac{9}{36} = \frac{1}{4} \quad \checkmark$$

$$A \cap C = B \cap C = A \cap B \cap C = \{(1,6), (2,5), (3,4)\} \Rightarrow$$

$$P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = \frac{3}{36} = \frac{1}{12}$$

$$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4}$$

$$P(A \cap C) = P(A) \cdot P(C) = \frac{1}{12}$$

$$P(B \cap C) = P(B) \cdot P(C) = \frac{1}{12}$$

$$\text{but } P(A \cap B \cap C) = \frac{1}{12} \neq P(A) \cdot P(B) \cdot P(C) = \frac{1}{24}$$

Therefore, events A, B, and C are pairwise independent, but they are not mutually independent!