ECE544: Fault-Tolerant Computing & Reliability Engineering

Lecture #11–

Reliability Block Diagrams (RBD)

Instructor: Dr. Liudong Xing

Fall 2022

Administrative Issues (10/19, Wed.)

- Project meeting (in-person or virtual)
  - Due by **Oct. 28, Friday**
- Today's topics
  - Midterm exam solution discussion
  - Midsemester survey
  - Lecture#11 (RBD)

Review of Lecture #9

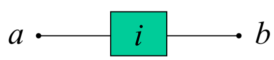
- Fault tree is not a tree in the graph-theoretic sense; it provides a logical framework for expressing combinations of component failures that can lead to system failure
- Top-down construction of fault trees provides a systematic method for analyzing and documenting the potential causes of system failure
- Qualitative analysis of fault trees based on cutsets can identify the single-point failures and system vulnerability
- Quantitative analysis of fault trees using cutsets
  - Inclusion/Exclusion (I/E)
  - Sum of Disjoint Products (SDP)
  - SDP is more efficient than I/E

Topics

- Reliability block diagrams (RBD)
  - Basic concepts
  - Series structure vs. parallel structure
  - RBD vs. Fault Trees
  - Path-sets vs. cut-sets

Reliability Block Diagrams (RBD)

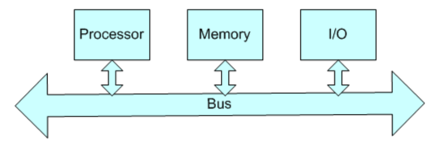
- A RBD is a **success-oriented** network describing the function of the system
  - A separate RBD is established for each system function if the system has more than one function
- Each component *i* of the system is represented by a block in the RBD

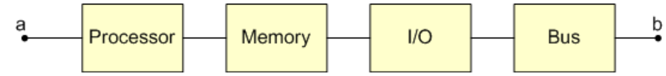


  - Component *i* is functioning if there is a connection between end points *a* and *b*

RBD: Example

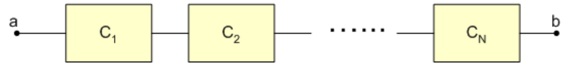
- Example: system is functioning *iff* the processor, memory, I/O and bus are all operational





Series Structure (1)

- A system that is functioning *iff* each component of the system is functioning
- A system containing no redundancy
- RBD of a general series system



Series Structure (2): Reliability

- System reliability
  - $C_{iw}(t)$: the event that component C_i is working properly at time t
  - $R_i(t)$: reliability of component C_i at time t
  - $R_{series}(t)$: reliability of the series system

$$R_{series}(t) = P\{C_{1w}(t) \cap C_{2w}(t) \cap \ldots \cap C_{Nw}\}$$

$C_{iw}(t)$s are independent

$$R_{series}(t) = R_1(t)R_2(t)\ldots R_N(t)$$

or

$$R_{series}(t) = \prod_{i=1}^{N} R_i(t)$$

Series Structure (3)

- A special case: assume an exponential failure rate λ_i for each component i in series

$$R_{series}(t) = R_1(t)R_2(t)\ldots R_N(t)$$

$$R_{series}(t) = e^{-\lambda_1 t}e^{-\lambda_2 t}\ldots e^{-\lambda_N t}$$

$$R_{series}(t) = e^{-\sum_{i=1}^{N}\lambda_i t}$$

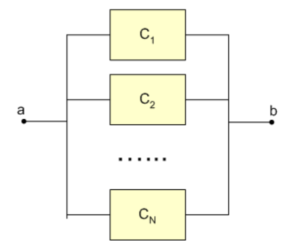
- Define λ_{system}, corresponding to the failure rate of the series system:

$$\lambda_{system} = \sum_{i=1}^{N}\lambda_i$$

Parallel Structure (1)

- A system that is functioning *if* at least one of its N components is functioning

- RBD of a general parallel system



Parallel Structure (2): Reliability

- System reliability
  - $C_{if}(t)$: the event that component C_i in the parallel system has failed at time t (independently)
  - $Q_i(t)/R_i(t)$: unreliability/reliability of component C_i at time t
  - $Q_{parallel}(t)/R_{parallel}(t)$: unreliability/reliability of the parallel system

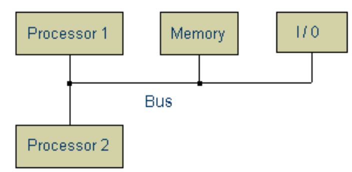
$$Q_{parallel}(t) = P\{C_{1f}(t) \cap C_{2f}(t) \cap \ldots \cap C_{Nf}(t)\}$$

$$Q_{parallel}(t) = Q_1(t)Q_2(t)\ldots Q_N(t) = \prod_{i=1}^{N} Q_i(t)$$

$$R_{parallel}(t) = 1.0 - Q_{parallel}(t) = 1.0 - \prod_{i=1}^{N} Q_i(t) = 1.0 - \prod_{i=1}^{N} (1.0 - R_i(t))$$

Series and Parallel Systems

- In practice, many systems are typically combinations of series and parallel structures
- Example:



  - System is operational *iff* 1 processor, memory, I/O and bus are functioning
  - Find RBD?
  - Find the system reliability given that
    - Two processors have the same reliability $R_p(t)$
    - Memory: $R_m(t)$
    - I/O: $R_{IO}(t)$
    - Bus: $R_b(t)$

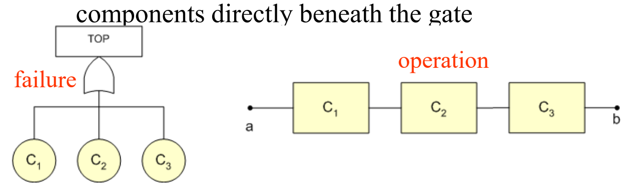
Topics

- Reliability block diagrams
  - √ Basic concepts
  - √ Series structure vs. parallel structure
  - – **RBD vs. Fault Trees**
  - – Path-sets vs. cut-sets

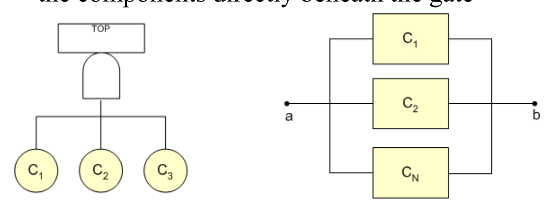
RBD vs. Fault Trees (I)

- Success oriented vs. failure oriented
- In terms of modeling capability

  RBD = Static/Traditional Fault Trees
- It's easy to convert a fault tree to a RBD,
  - Starting from the TOP event and replacing the gates successively
  - OR-gates are replaced by series structures of the components directly beneath the gate



  - AND-gates are replaced by parallel structures of the components directly beneath the gate

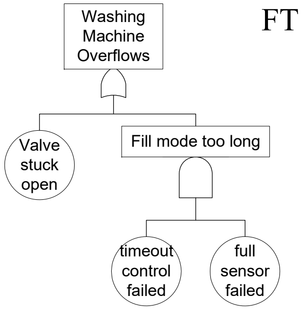


- And vice versa!

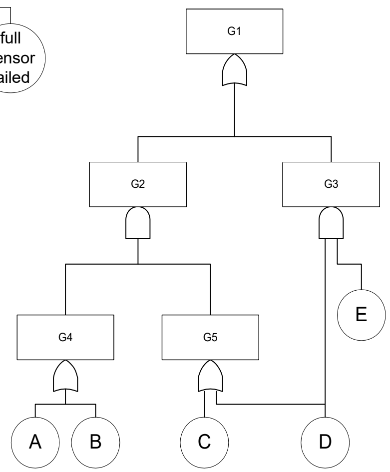
Hands-On Problems

- Find the RBD for the following fault trees (from L#9):

FT1



FT2



Topics

- Reliability block diagrams
  - √ Basic concepts
  - √ Series structure vs. parallel structure
  - √ RBD vs. Fault Trees
  - – **Path-sets vs. cut-sets**

Path-sets vs. Cut-sets (1)

- **Path-sets**: a set of components which by functioning ensures that the system is functioning
- **Minimal path-sets**: a path set is minimal if it cannot be reduced without losing its status as a path set

- **Cut-sets**: a set of components which by failing causes the system to fail
- **Minimal cut-sets**: a cut set is minimal if it cannot be reduced without losing its status as a cut set

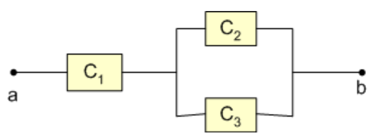
Path-sets vs. Cut-sets (2)

- The designer's point of view: consider a designer who wants to ensure that a system is functioning with the least possible design effort. What the designer needs is a list of the minimal path sets

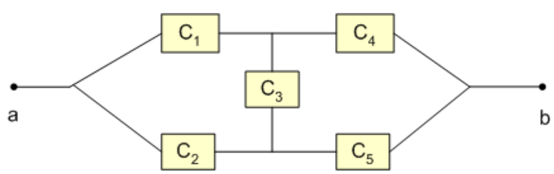
- The saboteur's point of view: consider a saboteur who wants to bring the system into a failed state with the least possible effort on his/her part. What the saboteur needs is a list of minimal cut sets

Hands-On Problems

- Find the minimal path-sets, and minimal cut-sets for the following systems:
- RBD-Ex1:



- RBD-Ex2: a bridge structure

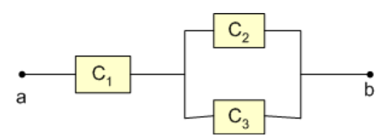


Another Way to Generate Minimal Path-Sets

- Fault tree → dual fault tree or

  RBD → fault tree → dual fault tree
  - The dual fault tree is obtained by replacing all AND gates in the original fault tree with OR gates, and vice versa; and letting the events in the dual fault tree be complements of the corresponding events in the original fault tree
- Apply the same procedure (top-down algorithm) as was described for generating minimal cutsets to the dual fault tree → minimal path sets!

- Example: Find the minimal path sets for RBD-Ex1 using the dual fault tree approach



Quantitative Analysis based on Minima Path-sets

- System reliability is the probability that all of the basic events in one or more minimal path-sets occur

$$\textbf{System Reliability} = \Pr\{\bigcup_{i=1}^{n} P_i\}$$

Quantitative Analysis based on Minima Cut-sets (Review)

- System failure is the probability that all of the basic events in one or more mincuts occur

$$Pr\{System \quad Failure\} = Pr\{\bigcup_i C_i\}$$

- Inclusion/exclusion

$$\begin{aligned} Pr\{\bigcup_{i=1}^{n} C_i\} &= \sum_{i=1}^{n} Pr\{C_i\} \\ &- \sum_{i<j} Pr\{C_i \bigcap C_j\} \\ &+ \sum_{i<j<k} Pr\{C_i \bigcap C_j \bigcap C_k\} \\ &\mp \cdots \\ &\pm Pr\{\bigcap_{i=1}^{n} C_i\} \end{aligned}$$

- Sum of disjoint product

$$Unreliablity = \Pr\{\bigcup_{i=1}^{n} C_i\}$$

$$= P(C_1) + P(\overline{C_1}C_2) + P(\overline{C_1}\,\overline{C_2}C_3) + \ldots + P(\overline{C_1}\,\overline{C_2}\,\overline{C_3}\ldots\overline{C_{n-1}}C_n)$$

Quantitative Analysis based on Minima Path-sets (Cont'd)

$$\textbf{System Reliability} = \Pr\{\bigcup_{i=1}^{n} P_i\}$$

- Inclusion/exclusion

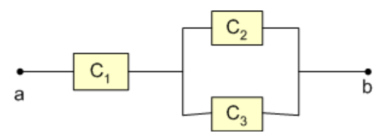
$$\text{Re}\,liablity = \Pr\{\bigcup_{i=1}^{n} P_i\} = \sum_{i=1}^{n} \Pr\{P_i\}$$
$$- \sum_{i<j} \Pr\{P_i \cap P_j\}$$
$$+ \sum_{i<j<k} \Pr\{P_i \cap P_j \cap P_k\}$$
$$\mp \ldots\ldots$$
$$\pm \Pr\{\bigcap_{j=1}^{n} P_j\}$$

- Sum of disjoint product

$$\text{Re}\,liablity = \Pr\{\bigcup_{i=1}^{n} P_i\}$$
$$= P(P_1) + P(\overline{P_1}P_2) + P(\overline{P_1}\,\overline{P_2}P_3) + \ldots + P(\overline{P_1}\,\overline{P_2}\,\overline{P_3}\ldots\overline{P_{n-1}}P_n)$$

Hands-On Problem

- For the RBD called RBD-Ex1, use the following four methods to calculate the system reliability
  - I/E based on minimal cut-sets
  - SDP based on minimal cut-sets
  - I/E based on minimal path-sets
  - SDP based on minimal path-sets



- Assume that the probability of occurrence for each of the basic events (i.e., the failure probability of the component C_i) is:

$\Pr\{C_1\} = 0.1$, $\Pr\{C_2\} = 0.2$, $\Pr\{C_3\} = 0.3$

Summary of Lecture #11

- A RBD is a **success-oriented** network describing the function of the system
- In terms of modeling capability, RBD is equivalent to the static/traditional fault trees, and they can be converted into each other easily
- Path-sets and cut-sets can be generated from both RBD and fault trees
- I/E and SDP can be applied to the quantitative analysis based on both path sets and cut sets

Next topic:

Binary decision diagrams (BDD)