ECE 454/544: Fault-Tolerant Computing & Reliability Engineering

Lecture #13–

Sensitivity Analysis

Instructor: Dr. Liudong Xing

Fall 2022

Administrative Issues
(10/31, Monday)

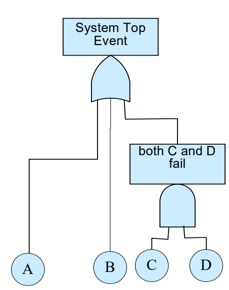
- Homework#5 due **Today**
- Homework#6 assigned
  - Due by **Nov. 7, Monday**
- Project final report
  - Due by **Nov. 30, Wednesday**
  - Please check out the Report Guidelines for requirements

Review of Lecture #12

- BDD is a DAG based on Shannon's decomposition
- BDD can be used to efficiently and accurately solve the combinatorial reliability models without the use of cutsets
- The size of the BDD depends heavily on the input variable ordering

Review Questions

- Generate the binary decision diagram (BDD) using the ordering of A<B<C<D.
- Given that all the components fail with a fixed failure probability of 0.1, computer the reliability of the entire system.
- Given that all the components fail with a constant failure rate 0.1/year, computer the reliability of the entire system at time t=2 years.



Agenda

- Component sensitivity/importance analysis
  - Basic concepts
  - Different importance measures
  - Evaluation of importance measures

5

Basic Concepts (1)

- System reliability depends on the reliability of its components
- Some components can be more critical than others in contributing to the entire system reliability.
- In 1969, Birnbaum introduced the concept of component importance measures
  - provide information concerning a component's contribution to the entire system reliability.
  - parameters that measure the change in system unreliability resulting from a change in component failure probability
  - measure the sensitivity of the system unreliability to the component failure parameters

6

Basic Concepts (2)

- Improvement Oriented - Sensitivity analysis helps identify which components contribute most to the system reliability and thus they will be good candidates for efforts leading to improving system reliability
  - Given limited resources such as a fixed engineering budget, how can the entire system reliability be best improved?
  - "How does a change in one component affect the entire system?"

- Maintenance Oriented - Sensitivity analysis helps identify the component that has the largest probability of being the cause of system failure → set up a repairperson's checklist

Two Classes of Importance Measures

- Structure-importance / deterministic-importance measures
  - the importance of a component to the system operation can be assessed by virtue of its position in the fault tree structure, without considering the reliability of the component

- Reliability-importance / probabilistic-importance measures

Structure Importance – An Example

- For a component *i*, we define a critical state as a state for the remaining *n-1* components such that the failure of component *i* causes the system to go from a working state to a failed state, given that the system contains *n* components

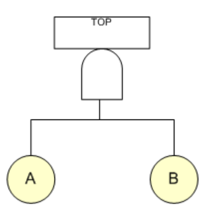
- Define a structure importance measure for component *i*:

$$I_i = \frac{\textbf{number of critical system states for component i}}{\textbf{total number of states for the (n - 1) remaining components}}$$

9

Structure Importance – An Example

- Example



To evaluate I_A:

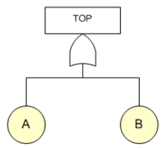
States for remaining components	Critical state for A
($\bar{B}$) B not failed	NO
(B) B failed	YES

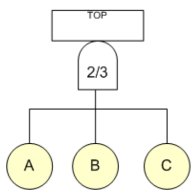
$$I_A = \frac{\textbf{number of critical states}}{\textbf{total number of states for other components}} = \frac{1}{2} = 0.5$$

10

Hands-On Problems (1)

- Find the structure importance I_A of component A for the following system fault tree models





Structure Importance

- Advantages:
  - can be used even if the component reliability is unknown or subject to changes

- Disadvantages:
  - cannot distinguish between components that occupy the similar structural positions but have drastically different reliabilities.
  - structure importance measures are of little practical importance due to not considering component failure parameters!

Two Classes of Importance Measures (Agenda)

✓ Structure-importance / deterministic-importance measures

- Reliability-importance / probabilistic-importance measures
  - consider both the position of the component and the component reliability to evaluate the component's importance
  - generally provide more information for generating the ranked list than the structure importance measures

Reliability Importance Measures

- Birnbaum's measure
- Criticality importance factor (measure)
- Diagnostic importance factor
- Conditional probability measure
- Improvement potential
- Risk achievement worth factor
- Risk reduction worth factor
- Etc.

Birnbaum's Measure

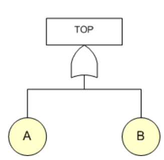
- Also known as criticality function, defined as a partial derivative of the system unreliability with respect to the probability of failure of the component

$$I^{BM}(e) = \frac{\partial \Pr\{S\}}{\partial \Pr\{e\}} = \frac{\partial U_{sys}}{\partial q_e}$$

  - measures the sensitivity of the system unreliability to changes in failure probability of the component

Hands-On Problem (2)

- Example
  - Consider the fault tree of a two-component series system, find the $I^{BM}(A)$ and $I^{BM}(B)$?



Birnbaum's Measure (Cont'd)

- A second method for defining Birnbaum's measure
  - The difference in the unreliability of the system with component i failed and the unreliability of the system with component i functioning

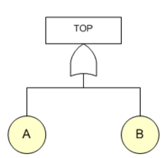
$$I^{BM}(e) = Q(1_i, q) - Q(0_i, q)$$

  - q: a vector of q_i
  - $Q(q)$: system unreliability
  - $(1_i, q)$: a probability vector with its ith entry equal to 1
  - $(0_i, q)$: a probability vector with its ith entry equal to 0

17

Hands-On Problem (3)

- Consider the fault tree of a two-component series system, find the $I^{BM}(A)$ and $I^{BM}(B)$?



18

Other Measures

- Conditional probability (CP)

$$I^{CP}(e) = \Pr\{S \mid e\} = \frac{\Pr\{S \cap e\}}{\Pr\{e\}} = \frac{\Pr\{S \cap e\}}{q_e}$$

- Diagnostic importance factor (DIF)
  - Gives the fraction of the system unreliability that involves the failure of the component *e*

$$I^{DIF}(e) = \Pr\{e \mid S\} = \frac{\Pr\{S \cap e\}}{\Pr\{S\}} = \frac{\Pr\{S \cap e\}}{U_{sys}}$$

- Criticality importance factor (CIF)
  - Gives the probability that component *e* has caused system failure, given that the system is failed at time t

$$I^{CIF}(e) = \frac{\Pr\{e\}}{\Pr\{S\}} I^{BM}(e) = \frac{q_e}{U_{sys}} I^{BM}(e)$$

Other Measures

- Risk achievement worth (RAW)

$$I^{RAW}(e) = \frac{\Pr\{S \mid e\}}{\Pr\{S\}} = \frac{I^{CP}(e)}{\Pr\{S\}} = \frac{\Pr\{S \cap e\}}{U_{sys} q_e}$$

  - Also called risk increase factor
  - Measures the increase in system unreliability assuming the worst case of the failure of component
- Risk reduction worth (RRW)

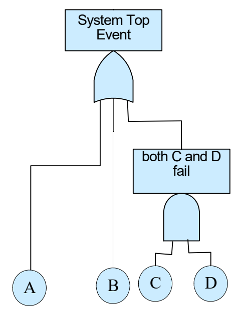
$$I^{RRW}(e) = \frac{\Pr\{S\}}{\Pr\{S \mid \bar{e}\}} = \frac{U_{sys}(1 - q_e)}{\Pr\{S \cap \bar{e}\}}$$

  - Also called risk decrease factor
  - Measures the decrease of the risk by increasing component reliability
- Improvement potential (IP)
  - Measures how much the system reliability increases if component *e* is replaced by a perfect component (q_e=0)

$$I^{IP}(e) = I^{BM}(e)\Pr\{e\} = I^{BM}(e) q_e = I^{CIF}(e) U_{sys}$$

Hands-On Problems (4)

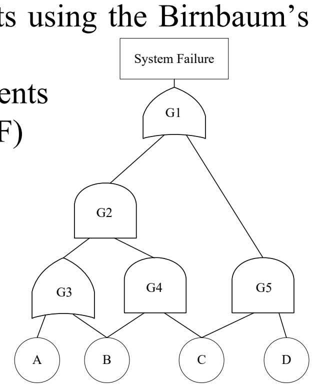
- For the system in the following fault tree mode, find the following measures for component A. Assume the component failure probabilities are: A (0.02), B (0.04), C (0.06), D (0.05)
  - Birnbaum's measure
  - Criticality importance factor (measure)
  - Diagnostic importance factor
  - Conditional probability measure
  - Improvement potential
  - Risk achievement worth factor
  - Risk reduction worth factor



Hands-On Problem (5)

Consider a system whose failure is modeled by the following fault tree.

a) Generate the system BDD
b) All the system components fail independently. Assume each component has the same fixed failure probability of 0.1. Determine the reliability of the entire system.
c) Rank importance of all system components using the Birnbaum's measure
d) Rank importance of all the system components using the diagnostic importance factor (DIF)



References

1. Y. Dutuit and A. Rauzy, "Efficient algorithm to assess component and gate importance in fault tree analysis", *Reliability Eng and System Safety*, vol. 72, 2001, pp 213-222.

2. M. Rausand and A. Hoyland, *System Reliability Theory: Models and Statistical Methods*, 2003, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons.

3. L. Xing, Maintenance-Oriented Fault Tree Analysis of Component Importance, Proceedings of The 50th Annual Reliability and Maintainability Symposium (RAMS), Los Angeles, CA, January 2004

4. A. Anne, *Implementation of Sensitivity Measures for Static and Dynamic Subtrees in DIFtree,* 1997, M.S. Thesis, University of Virginia.

5. P. M. Frank, *Introduction to System Sensitivity*, 1978, Academic Press.

23

Summary of Lecture #13

- Component sensitivity analysis measures the sensitivity of the system unreliability to the component failure parameters
  - Improvement Oriented: helps identify which components contribute most to the system reliability and thus they will be good candidates for efforts leading to improving system reliability, e.g.: Birnbaum's measure, improvement potential
  - Maintenance Oriented: helps identify the component that has the largest probability of being the cause of system failure → set up a repairperson's checklist, e.g.: criticality importance factor, diagnostic importance factor, Fussel-Vesley measure

Things to Do

- Homework & Project

24