ECE544: Fault-Tolerant Computing & Reliability Engineering
(Fall 2022)

Homework #4 Solution **(55 points)**

Problems:

1. **(20 points)** Moon Systems, a manufacturer of scientific workstations, produces its Model 13 System at sites S1, S2, S3; 20% at S1, 35% at S2, and the remaining 45% at S3. The probability that a Model 13 System will be found defective upon receipt by a customer is 0.01 if it is shipped from site S1, 0.06 if from S2, and 0.03 if from S3.
   a. What is the probability that a Model 13 System selected at random at a customer location will be found defective? (10 points)
   b. Suppose a Model 13 System selected at random is found to be defective at a customer location. What is the probability that it was manufactured at site S2? (10 points)

Solution:
Define events:
E_1: The Model 13 System is produced at site S1
E_2: The Model 13 System is produced at site S2
E_3: The Model 13 System is produced at site S3
E: Model 13 System selected at random at a customer location is found defective

we have:

$P(E_1)=0.2 \quad P(E_2)=0.35 \quad P(E_3)=0.45$

$P(E|E_1)=0.01 \quad P(E|E_2)=0.06 \quad P(E|E_3)=0.03$

(a) $P(E) = \sum_{i=1}^{3} P(E|E_i)\cdot P(E_i) = 0.2\times 0.01 + 0.35\times 0.06 + 0.45\times 0.03 = 0.0365$

(Based on "Total Probability Law")

(b) $P(E_2|E) = \dfrac{P(E_2 E)}{P(E)} = \dfrac{P(E|E_2)\cdot P(E_2)}{P(E)} = \dfrac{0.06\times 0.35}{0.0365}$

$= 0.57534$

2. **(20 points)** A component with time to failure *T* has constant failure rate $\lambda = 2.5 * 10^{-5}$/hour

   a. (5 points) Determine the probability that the component survives a period of 2 months without failure (Assume each month has 30 days).

   b. (5 points) Find the MTTF of the component.

   c. (5 points) Find the probability that the component survives its MTTF.

   d. (5 points) Suppose the component has been functioning without failures during its first 3 months in operation. Find the probability that the component will survive another 2 months.

a. $t = 2$ months $= 2 \times 30 \times 24$ hours $= 1440$ hours

$$R(t) = e^{-\lambda t} = e^{-2.5\times10^{-5}\times1440} = e^{-3.6\times10^{-2}} = 0.9646$$

b. Constant failure rate means T is exponentially distributed.

So, $MTTF = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda} = \frac{1}{2.5\times10^{-5}}$ hours

$\therefore MTTF = 4\times10^{4}$ hours

c. $R(t=MTTF) = e^{-\lambda t} = e^{-2.5\times10^{-5}\times4\times10^{4}} = e^{-1} \doteq 0.3679$

d) Due to the memoryless property, the answer is the same as part a), that is 0.9646

3. **(15 points)** A component may fail due to two different causes, A and B. It has been shown that the time to failure T_A caused by A is exponentially distributed with density function $f_A(t) = \lambda_A e^{-\lambda_A t}$ for t>=0, while the time to failure T_B caused by B is exponentially distributed with density function $f_B(t) = \lambda_B e^{-\lambda_B t}$ for t>=0.
   a. (10 points) Describe the rationale behind using $f(t) = pf_A(t) + (1-p)f_B(t)$ as the probability density function for the time to failure T of the component
   b. (5 points) Explain the meaning of p in this model

a. Since there are only 2 different causes that can lead to the failure of the component, according to "Total probability Theorem", we have:

$$f(t) = f(t \mid \text{cause} = A) \cdot P\{\text{cause} = A\} + f(t \mid \text{cause} = B) \cdot P\{\text{cause} = B\}$$

$$= f_A(t) \cdot P\{\text{cause} = A\} + f_B(t) \cdot P\{\text{cause} = B\}$$

and $P\{\text{cause} = A\} + P\{\text{cause} = B\} = 1$.

if we let $P\{\text{cause} = A\} = p$, then

$$f(t) = p \cdot f_A(t) + (1-p) \cdot f_B(t)$$

b. p denotes the probability that the failure of the component is caused by A.