Hands-On Problem (1)

- A rotary pump has a constant failure rate $\lambda=10^{-4}$/hour [Rausand03]
  - What is the probability that the pump survives 1 month in continuous operation?
  - What is the MTTF of the pump?
  - Suppose the pump has been functioning without failure during its first 2 months in operation. What is the probability that the pump will fail during the next month?

1. $\lambda = 10^{-4}/hr$

$$R(t = 1\ \text{month}) = e^{-\lambda t} = e^{-10^{-4} * 30 * 24}$$

$$= 0.9305$$

2. $$MTTF = \frac{1}{\lambda} = \frac{1}{10^{-4}} = 10^{4}\ hrs$$

3. Memoryless property

$$P\{T \le t_1 + t_2 \mid T > t_1\}$$ $t_1 = 2$ months, $t_2 = 1$ month

$$= P\{T \le t_2\}$$

$$= 1 - e^{-\lambda t_2} = 1 - 0.9305 = 0.0695$$

Hands-on Problem (2)

Mixture of Exponential Distributions

- Assume that the same type of items are produced at two different plants. The items are independent and have constant failure rates. The production process is slightly different at two plants and items will have different failure rates (λ_i: for items from plant i). The items are mixed up before being sold. A fraction p is coming from plant 1, and the rest is from plant 2. If we (1) pick one item at random, what is the reliability function of this item?

(2) What is the MTTF of the item?

(1) Total probability Law

$$R(t) = p \cdot R_1(t) + (1-p) \cdot R_2(t)$$

$$= p \cdot e^{-\lambda_1 t} + (1-p)\, e^{-\lambda_2 t}$$

(2) $$MTTF = \int_0^\infty R(t)\,dt$$

$$= \int_0^\infty p e^{-\lambda_1 t} dt + \int_0^\infty (1-p) \cdot e^{-\lambda_2 t} dt$$

$$= p \frac{1}{\lambda_1} + (1-p) \frac{1}{\lambda_2}$$

$$= \frac{p}{\lambda_1} + \frac{1-p}{\lambda_2}$$