2. A component with time to failure *T* has failure rate $z(t) = 2.0*10^{-6}\, t$/hour for $t>0$
   a. Determine the probability that the component survives 200 hours
   b. Determine the probability that a component, which is functioning after 200 hours, is still functioning after 400 hours

   Note that the failure rate in this problem is not constant, but a function of time *t*.

a.

Given $z(t) = 2.0\times10^{-6}t$,

a. $R(t=200) = e^{-\int_0^t z(u)du} = e^{-\int_0^{200} ku\,du} = e^{-k\times\frac{u^2}{2}\Big|_0^{200}}$ $(k = 2\times10^{-6})$

$= e^{-k\times2\times10^4} = e^{-2\times10^{-6}\times2\times10^4} = e^{-4\times10^{-2}} \approx 0.9608$

b.

$\therefore\ P_r\{T>400 \mid T>200\} = \dfrac{P_r\{T>400,\ T>200\}}{P_r\{T>200\}} = \dfrac{P_r\{T>400\}}{P_r\{T>200\}}$

$= \dfrac{R(400)}{R(200)}$

$R(t) = e^{-\int_0^t z(u)du} = e^{-\int_0^t ku\,du} = e^{-k\times\frac{t^2}{2}} = e^{-10^{-6}t^2}$

$\therefore\ R(400) = e^{-10^{-6}\times400^2} = e^{-0.16}$

$R(200) = e^{-10^{-6}\times200^2} = e^{-0.04}$

So, $P_r\{T>400 \mid T>200\} = \dfrac{e^{-0.16}}{e^{-0.04}} = e^{-0.12} \approx 0.8869$

For your reference: the MTTF is:

$MTTF = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-\int_0^t z(u)du}\,dt$

$= \int_0^\infty e^{-\int_0^t ku\,du}\,dt = \int_0^\infty e^{-k\times\frac{t^2}{2}}\,dt$

$= \int_0^\infty e^{-10^{-6}\times t^2}\,dt \approx 886$ hours