ECE544 Fault-Tolerant Computing & Reliability Engineering
Solution to Midterm Exam Sample Questions (F'22)

1. Considers the following two competing fault-tolerant designs that can tolerate 1 fault:
   Design #1 (2-out-of-3: G system): consists of three identical components working concurrently; the system is Good (i.e., functioning) if 2 or more out of the three components are working correctly.
   Design #2 (3-out-of-4: G system): consists of four identical components working concurrently; the system is functioning if three or more out of the four components are working correctly.
   Assume all components used in the two designs have a fixed reliability of p=0.7 for a prescribed time. Which design is more reliable? Please justify your answer by computing the reliability of each design.

$p = 0.7$

$$R_{D1} = \sum_{i=2}^{3} C_3^i \, p^i (1-p)^{3-i} = 3p^2(1-p) + p^3$$
$$= 0.784$$

$$R_{D2} = \sum_{i=3}^{4} C_4^i \, p^i (1-p)^{4-i} = 4p^3(1-p) + p^4$$
$$= 0.6517$$

$R_{D1} > R_{D2}$ So Design #1 is more reliable!

2. For a 4-bit data word (d0, d1, d2, d3)=(0111), determine

a). if the word (v0,v1,v2,v3,v4,v5,v6)=(0100011) is *its* valid non-separable (7,4) cyclic code word with the generator polynomial being $G(X)=1+x+x^3$. Justify your answer.

b). if the word (v0,v1,v2,v3,v4,v5,v6)=(0110011) is *its* valid Hamming SEC code word or not. Justify your answer.

ⓐ $V(x) = x + x^5 + x^6$. If valid, $V(x) = D(x) * G(x)$!

$$x^3 + x^2 + x = D(x)$$

$$x^3+x+1 \overline{)\, x^6 + x^5 + x}$$

$$x^6 + x^4 + x^3$$

$$x^5 + x^4 + x^3 + x$$

$$x^5 + x^3 + x^2$$

$$x^4 + x^2 + x$$

$$x^4 + x^2 + x$$

$$0 = R(x)$$

Since $R(x)=0$, so it's valid!

ⓑ $2^k - 1 \geq m + k$ $m=4 \Rightarrow k=3$

	7	6	5	4	3	2	1
position#	111	110	101	100	011	010	001
	D_3	D_2	D_1	C_4	D_0	C_2	C_1

$c_1 = d_0 \oplus d_1 \oplus d_3 = 0 \oplus 1 \oplus 1 = 0$

$c_2 = d_0 \oplus d_2 \oplus d_3 = 0 \oplus 1 \oplus 1 = 0$

$c_4 = d_1 \oplus d_2 \oplus d_3 = 1 \oplus 1 \oplus 1 = 1$

Valid SEC codeword is:

1 1 1 1 0 0 0

v_6 v_0

therefore the give word 0110011 is not valid!

3. A bulb with an exponential time-to-failure distribution has the mean time to failure of 10000 hours.
   a. Determine the failure rate of the bulb.
   b. Find the probability that the bulb will survive its MTTF in continuous operation.
   c. Determine the probability that the bulb will fail within 15000 hours, when you know that the bulb was functioning at 5000 hours.
   d. Find the mean residual life (MRL) of the bulb at age t=70000hours.

$MTTF = 10000 = 10^4$ hours

a) $MTTF = \frac{1}{\lambda}$

$\Rightarrow \lambda = \frac{1}{MTTF} = 10^{-4}/hrs$

b) $R(t = MTTF) = e^{-\lambda t} = e^{-10^{-4} \times 10^4} = e^{-1} = 0.367879$

c) $F(t = 10000 hrs) = 1 - R(t = 10000 hrs)$

$= 1 - 0.367879 = 0.632121$

Based on "memoryless property"

or

$Pr\{T < 15000 \mid T \geq 5000\} = \frac{Pr\{5000 \leq T < 15000\}}{Pr\{T \geq 5000\}}$

$= \frac{F(15000) - F(5000)}{R(5000)} = \frac{1 - e^{-\lambda \times 15000} - 1 + e^{-\lambda \times 5000}}{e^{-\lambda \cdot 5000}}$

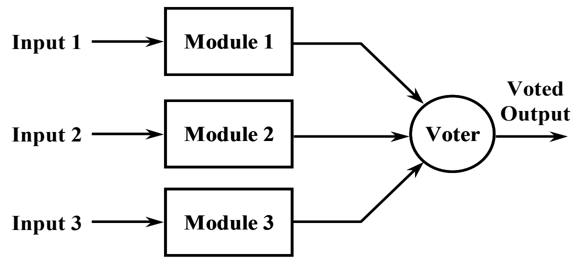
$= \frac{e^{-\lambda 5000} - e^{-\lambda 15000}}{e^{-\lambda 5000}} = \frac{e^{-\lambda 5000}(1 - e^{-\lambda 10000})}{e^{-\lambda 5000}}$

$= 1 - e^{-\lambda 10000} = F(t = 10000) = 0.632121$

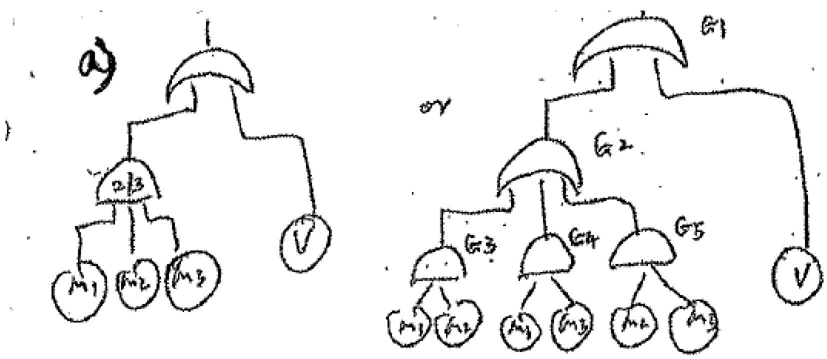
d) MRL at age/time $t = 70000$ hrs?
For exponential distribution, $MRL = MTTF = 10000$ hrs.

4. Consider a Triple Module Redundancy (TMR) system as shown in the following figure. Suppose all the three modules fail independently and exponentially with the same constant failure rate λ=0.0001/hour. The voter fails with the fixed probability of 0.001.

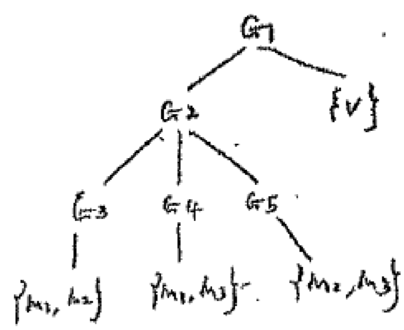
a. Show the fault tree model of this TMR system.
b. Find all the minimal cut sets.
c. Find the system unreliability for time t=1000 hours



Note: for simplicity, you may use M1, M2, M3, and V to denote the three modules and the voter in your answers.



b)



4 mincuts: C1={V}, C2={M1, M2}, C3={M1, M3}, C4={M2, M3}

c) You may do the inclusion-exclusion or SDP evaluation based on three minimal path sets or based on the four mincuts, or through the following method.

$R_{sys} = R_M * R_V$

$= \left[C_3^3 p_m^3 (1-p_m)^0 + C_3^2 p_m^2 (1-p_m)\right] * p_v$

$= \left[p_m^3 + 3 p_m^2 (1-p_m)\right] * p_v$

$= \left(3p_m^2 - 2p_m^3\right) * p_v$

$p_v = 1 - 0.001 = 0.999$ (reliability of the voter)

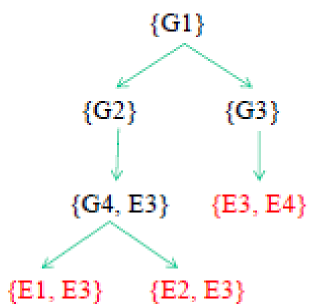
$p_M = e^{-\lambda t} = e^{-0.0001 * 1000} = e^{-0.1} = 0.9048$ (reliability of the module)

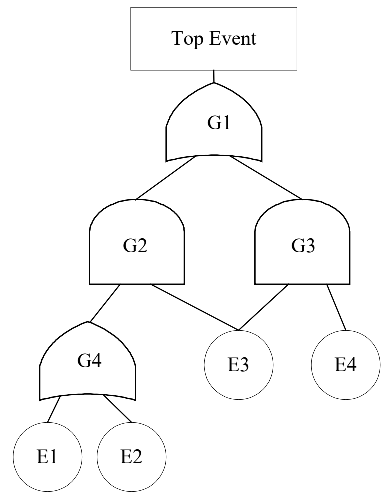
$R_{sys} = 0.9746 * 0.999$

$= 0.9736$

5. (Hands-on problem on Slide 35, Lecture #9) Consider the fault tree below modeling the failure of a system. Assume the occurrence probabilities of basic events are: Pr(E1)=0.1, Pr(E2)=0.05, Pr(E3)=0.01, Pr(E4)=0.02. Determine the system failure probability.

Cutsets generation:





I/E evaluation:

C_1={E1, E3}, C_2={E2, E3}, C_3={E3, E4}
Pr(E1)=0.1, Pr(E2)=0.05, Pr(E3)=0.01, Pr(E4)=0.02

Pr(C_1)=Pr(E1)Pr(E3)=0.001, Pr(C_2)=Pr(E2)Pr(E3)=0.0005
Pr(C_3)=Pr(E3)Pr(E4)=0.0002 -- *MC Failure Probabilities*

$$Q_{sys}=\Pr\{C_1\cup C_2\cup C_3\}=\sum_{i=1}^{3}\Pr(C_i)-\sum_{i<j}\Pr(C_i\cap C_j)+\Pr(C_1\cap C_2\cap C_3)$$
$$=\sum_{i=1}^{3}\Pr(C_i)-\Pr(C_1\cap C_2)-\Pr(C_1\cap C_3)-\Pr(C_2\cap C_3)+\Pr(C_1\cap C_2\cap C_3)$$
$$=0.0017-\Pr(E1E2E3)-\Pr(E1E3E4)-\Pr(E2E3E4)+\Pr(E1E2E3E4)$$
$$=0.0017-0.00005-0.00002-0.00001+0.000001$$
$$=0.001621$$

SDP evaluation:

C_1={E1, E3}, C_2={E2, E3}, C_3={E3, E4}
Pr(E1)=0.1, Pr(E2)=0.05, Pr(E3)=0.01, Pr(E4)=0.02
Pr(C_1)=0.001, Pr(C_2)=0.0005 Pr(C_3)=0.0002

$$Q_{sys}=\Pr(C_1)+\Pr(\overline{C_1}C_2)+\Pr(\overline{C_1C_2}C_3)$$
$$=P(E1)P(E3)+\Pr(\overline{E1E3}E2E3)+\Pr(\overline{E1E3}\,\overline{E2E3}E3E4)$$
$$=0.001+\Pr\left((\overline{E1}+\overline{E3})E2E3\right)+\Pr((\overline{E1}+\overline{E3})(\overline{E2}+\overline{E3})E3E4)$$
$$=0.001+\Pr(\overline{E1}E2E3)+\Pr(\overline{E1}\,\overline{E2}E3E4)$$
$$=0.001+0.9*0.05*0.01+0.9*0.95*0.01*0.02$$
$$=0.001621$$