# Maintenance-Oriented Fault Tree Analysis of Component Importance

Liudong Xing, Ph.D., University of Massachusetts, Dartmouth

Key Words: Common-cause failure, Fault tree, Improvement-oriented, Maintenance-oriented, Reliability importance

*SUMMARY & CONCLUSIONS*

In this paper we investigate and compare a set of existing component importance measures and select the most informative and appropriate one for guiding the maintenance of the system. Efficient methods to compute the selected measure are presented. An important concern in the traditional fault tree reliability analysis, common-cause failure, is also addressed in the component importance analysis using the selected measure. A simple example is designed and analyzed to show the selection process.

## 1. *INTRODUCTION*

Reliability analysis is a key component in the design, analysis, and tuning of computer-based systems. However, reliability analysis tells only part of the story. Follow-up questions such as "How does a change in one component affect the entire system?", "Given limited resources such as a fixed engineering budget, how can the entire system reliability be best improved?" have to be answered. These and similar questions are best answered using the results of importance analysis (also called sensitivity analysis, Ref. 1), which is referred to as improvement-oriented importance analysis (IO-IA) in this paper. The IO-IA helps identify which components contribute most to the system reliability and thus they will be good candidates for efforts leading to improving system reliability. For this purpose, various ways of defining such component importance have been proposed. A summary of them can be found in Ref. 2. In this paper, we consider another application of component importance analysis, that is, in the assistance of the system maintenance. For example, the measure would, by means of a list, tell the repairperson in which order to check the component that may have caused the system failure. Ideally speaking, the maintenance-oriented importance analysis (MO-IA) will rank the component whose repair will hasten the system recovery the most, the highest. Due to the different purposes between the IO-IA and MO-IA, it is not feasible to apply directly the existing measures for IO-IA to perform the MO-IA. Therefore, one main task of this paper is to investigate and compare the existing importance measures, and then select the most informative and appropriate measure to perform the importance analysis for assisting system maintenance. Further, we explore efficient approaches to evaluate the resulted measure, considering the systems in the presence of common cause failures.

The paper is organized as follows. First we briefly review the mathematical definitions of the importance measures we consider in this paper. In section 3, we present an illustrating example system, which is designed to show the semantics of the importance measures of interest and the selection process. In section 4, we study and compare the experimental results generated from the interested measures and propose the most appropriate one for guiding system maintenance. In section 5, we present the basic algorithms to assess the selected measure. Finally, the method for incorporating the common-cause failures into the MO-IA using the selected measure is presented in section 6.

### 1.1 *Acronyms*

| | |
|---|---|
| BDD | Binary Decision Diagram |
| BM | Birnbaum's Measure |
| CC | Common Cause |
| CCE | Common-Cause Event |
| CCF | Common-Cause Failure |
| CIF | Criticality Importance Factor |
| CP | Conditional Probability |
| DIF | Diagnostic Importance Factor |
| IO-IA | Improvement-Oriented Importance Analysis |
| IP | Improvement Potential |
| MO-IA | Maintenance-Oriented Importance Analysis |
| RAW | Risk Achievement Worth |
| RI | Reliability Importance |
| RRW | Risk Reduction Worth |
| SI | Structure Importance |

### 1.2 *Notations*

| | |
|---|---|
| $e$ | A basic event in fault tree, or a component in the system |
| $I^{BM}(e)$ | Birnbaum's importance measure of component $e$ |
| $I^{CIF}(e)$ | Criticality importance factor of component $e$ |
| $I^{CP}(e)$ | Conditional probability measure of component $e$ |
| $I^{DIF}(e)$ | Diagnostic importance factor of component $e$ |
| $I^{IP}(e)$ | Improvement potential measure of component $e$ |
| $I^{RAW}(e)$ | Risk achievement worth factor of component $e$ |
| $I^{RRW}(e)$ | Risk reduction worth factor of component $e$ |
| $q_e$ | Unreliability of component e, i.e., $Pr\{e\}$ |
| $S$ | Structure function of the system fault tree model under study |
| $U_{sys}$ | Unreliability function of the system, i.e., $Pr\{S\}$ |

## 2. *BACKGROUND*

Two classes of component importance measures, structural-importance (SI) and reliability-importance (RI), have been proposed for the case where the support model is a fault tree. By using the SI measures, the importance of a component to the system operation can be assessed by virtue of its position in the fault tree structure, without considering the reliability of the component (Ref. 2). Thus, they can be used even if the component reliability is unknown or subject to changes. However, the SI measures cannot distinguish between components that occupy the similar structural positions but have drastically different reliabilities. On the other hand, the RI measures consider both the position of the component and the component reliability (Ref. 2), thus generally provide more information for generating the ranked list than the SI measures. In this paper, our study focuses on the RI measures.

We present seven of existing RI measures in this section. We recall their mathematical definitions as well as their possible physical interpretations. It is worth noticing that all of these importance measures depend on the time *t* at which the system and its components are observed. In the following, the time *t* is omitted although it is implicitly present in all of the definitions. *S* denotes the structure function of the fault tree model under investigation, and *e* denotes a basic event (a component failure) in the system fault tree.

### 2.1 *Conditional Probability (CP)*

The conditional probability measure, denoted by $I^{CP}(e)$, is defined as $I^{CP}(e)=Pr\{S|e\}$. According to the definition of conditional probabilities (Ref. 8), we have:

$$I^{CP}(e) = \Pr\{S \mid e\} = \frac{\Pr\{S \cap e\}}{\Pr\{e\}} = \frac{\Pr\{S \cap e\}}{q_e} \tag{1}$$

### 2.2 *Risk Achievement Worth (RAW)*

The risk achievement worth, denoted by $I^{RAW}(e)$, is defined as $I^{RAW}(e) = \frac{\Pr\{S \mid e\}}{\Pr\{S\}} = \frac{I^{CP}(e)}{\Pr\{S\}}$. By the definition of conditional probabilities, we have:

$$I^{RAW}(e) = \frac{I^{CP}(e)}{\Pr\{S\}} = \frac{\Pr\{S \cap e\}}{U_{sys} q_e} \tag{2}$$

$I^{RAW}(e)$ is also called Risk Increase Factor (Ref. 4). It measures the increase in system unreliability assuming the worst case of the failure of the component.

### 2.3 *Risk Reduction Worth (RRW)*

The risk reduction worth, denoted by $I^{RRW}(e)$, is defined as $I^{RRW}(e) = \frac{\Pr\{S\}}{\Pr\{S \mid \bar{e}\}}$. By the definition of conditional probabilities, we have:

$$I^{RRW}(e) = \frac{\Pr\{S\}}{\Pr\{S \mid \bar{e}\}} = \frac{U_{sys}(1 - q_e)}{\Pr\{S \cap \bar{e}\}} \tag{3}$$

$I^{RRW}(e)$ is also called Risk Decrease Factor (Ref. 4). It measures the decrease of the risk (system unreliability) by increasing the reliability of the component. It is argued in Ref. 4 that RRW measure may be used to select components that are the best candidates for efforts leading to improving system reliability.

### 2.4 *Diagnostic Importance Factor (DIF)*

The diagnostic importance factor, denoted by $I^{DIF}(e)$, is defined as $I^{DIF}(e)=Pr\{e|S\}$. According to the definition of conditional probabilities (Ref. 8), we have:

$$I^{DIF}(e) = \Pr\{e \mid S\} = \frac{\Pr\{S \cap e\}}{\Pr\{S\}} = \frac{\Pr\{S \cap e\}}{U_{sys}} \tag{4}$$

$I^{DIF}(e)$ gives the fraction of the system unreliability (or risk) that involves the failure of the component *e*.

### 2.5 *Birnbaum's Measure (BM)*

The Birnbaum's measure, denoted by $I^{BM}(e)$, is defined as

$$I^{BM}(e) = \frac{\partial \Pr\{S\}}{\partial \Pr\{e\}} = \frac{\partial U_{sys}}{\partial q_e} \tag{5}$$

$I^{BM}(e)$ is thus obtained by partial differentiation of the system unreliability with respect to the probability of failure of the component. That is, it measures the sensitivity of the system unreliability to changes in failure probability of the component (Ref. 7).

### 2.6 *Criticality Importance Factor (CIF)*

The criticality importance factor, denoted by $I^{CIF}(e)$, is defined as

$$I^{CIF}(e) = \frac{\Pr\{e\}}{\Pr\{S\}} I^{BM}(e) = \frac{q_e}{U_{sys}} I^{BM}(e) \tag{6}$$

The $I^{CIF}(e)$ is thus the probability that component *e* has caused system failure, given that the system is failed at time *t* (Ref. 7).

### 2.7 *Improvement Potential (IP)*

The improvement potential with respect to component e at time t, denoted by $I^{IP}(e)$, is defined as (Ref. 7)

$$I^{IP}(e) = I^{BM}(e)\Pr\{e\} = I^{BM}(e) q_e \tag{7}$$

It may also be expressed by the CIF measure $I^{CIF}(e)$ by using eq. (6):

$$I^{IP}(e) = I^{BM}(e) q_e = I^{CIF}(e) U_{sys} \tag{8}$$

It measures how much the system reliability increases if component *e* is replaced by a perfect component, that is, a component such that its failure probability $q_e=0$.

## 3. *AN EXAMPLE SYSTEM*

We design a simple system to illustrate the semantics of the importance measures presented in section 2 as well as the process for selecting the most informative measure for guiding the system maintenance. Figure 1 (a) shows the fault tree model of the example system; figure 1 (b) shows the corresponding series-parallel reliability block diagram. We design 8 sets of component failure parameters (table 1) in order to illustrate the effects of the following two factors on the different importance measures:

- The position of the component in the system structure
- The unreliability of the component in question

The last column of table 1 gives Pr{C ∩ D}, the equivalent unreliability of the parallel subsystem that is composed of components C and D. The example failure parameters are designed to cover the following five cases:

- Pr{C∩D} is less than unreliability of the most unreliable component in the series structure (A or B): Set I, II
- Pr{C∩D} is equal to unreliability of the most unreliable component in the series structure (A or B): Set III, IV
- Pr{C∩D} is between unreliability of the most unreliable component and unreliability of the most reliable component in the series structure (A or B): Set V, VI
- Pr{C∩D} is equal to the unreliability of the most reliable component in the series structure (A or B): Set VII
- Pr{C∩D} is greater than unreliability of the most reliable component in the series structure (A or B): Set VIII

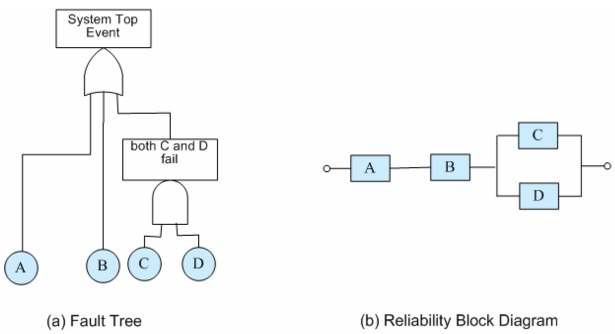


Figure 1: Example System Structure in Fault Tree and Reliability Block Diagram

| Comp. | A | B | C | D | C ∩ D |
|---|---|---|---|---|---|
| Set I | 0.02 | 0.04 | 0.06 | 0.05 | 3e-3 |
| Set II | 0.02 | 0.04 | 0.001 | 0.5 | 5e-4 |
| Set III | 0.02 | 0.04 | 0.1 | 0.2 | 0.02 |
| Set IV | 0.02 | 0.04 | 0.04 | 0.5 | 0.02 |
| Set V | 0.01 | 0.1 | 0.1 | 0.5 | 0.05 |
| Set VI | 0.02 | 0.04 | 0.06 | 0.5 | 0.03 |
| Set VII | 0.02 | 0.04 | 0.08 | 0.5 | 0.04 |
| Set VIII | 0.02 | 0.04 | 0.1 | 0.5 | 0.05 |

Table 1: Failure Parameters of the Example System

## 4. *INVESTIGATION & DISCUSSION*

Tables 2 – 9 present the results of performing component importance analysis for the example system (section 3) using the seven measures depicted in section 2. The last column of each table provides the ranking of the components. The example shows that the measures may lead to different rankings. This was to be expected, since the measures are defined differently.

### 4.1 *Observations*

The observations resulted from the experimental results in tables 2-9 include:

1. Both CP and RAW measures cannot distinguish between components that occupy similar positions in a series structure but have drastically different failure probabilities. This result is unreasonable because it is clear that the most unreliable component in the series structure should be ranked the highest in the checklist of the repairperson.
2. The experimental results for measures of RRW, CIF, and IP illustrate the fact that in a series structure, the component with the lowest reliability has the highest importance value; the components with the same equivalent failure probability have the same importance value. For example, in table 4, RRW, CIF, and IP measures give the same ranking of B>A=C=D, because the components in the series structure, that is, A, B, and C ∩ D, have the failure probabilities of 0.02, 0.04, and 0.02, respectively.
3. The experimental results for measures of RRW, CIF, and IP also show that all the components in a parallel structure have the same importance when we use the RRW, CIF or IP measure. This result seems reasonable because if a parallel structure fails, it will start functioning again irrespective of which of the components we repair.
4. The experimental results for measures of CP, RAW, and BM illustrate that irrespective of the failure probabilities of the components, the components A and B in the series structure always have higher ranking than components C and D in the parallel structure. This may induce misleading ordering in terms of guiding the system maintenance.
5. The experimental results for the BM measures show that for a component in a series structure, the more reliable it is, the lower ranking it has; however, for the component in a parallel structure, the more reliable it is, the higher ranking it has. From the maintenance point of view, this result seems unreasonable, because the most unreliable component in a series or a parallel structure should always be checked first.
6. Among the seven measures, the DIF measure is the most dynamic and responsive one in the sense that the ranking using the DIF can change according to the changes in the component reliabilities. Moreover, the importance analysis using DIF measure can always result in a deterministic ranking of the components. In other words, the components that occupy similar structural positions (for both series and parallel structures) but have different reliabilities will be ranked differently.
7. To support our later conclusions, it is necessary for us to examine the DIF measure further. Consider the parameter sets I and II in table 1, both sets share the same characteristic that the equivalent unreliability of the parallel subsystem (3e-3) is less than the unreliability of A (0.02)) or unreliability of B (0.04). Hence, the RRW, CIF, and IP measures produce the ranking of B>A>C=D. However, the DIF measure gives the ranking of

 0-7803-8215-3/04/$17.00 

B>A>C>D when the parameter set I is used, the ranking of B>D>A>C when the parameter set II is used. The DIF measure provides more information for generating the ranked list than the RRW, CIF, and IP measures because it can distinguish between components that occupy similar positions in a parallel structure but have different failure probabilities: the more unreliable the component is, the higher ranking it has. Also, the DIF measure can account for the effects of the component that has drastically different unreliability from others. For examples, the component D in set II is much more unreliable than the other components (A, B, C), though the equivalent unreliability of the parallel subsystem the component D is belonging to is the lowest, the component D is ranked higher than the component A. This is reasonable. Similar conclusions can be obtained by comparing the experimental results of set II and IV, or set V and VI.

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.106 | 0.116 | A=B>D>C |
| RAW | 16.123 | 16.123 | 1.713 | 1.865 | A=B>D>C |
| RRW | 1.446 | 2.704 | 1.048 | 1.048 | B>A>C=D |
| DIF | 0.323 | 0.645 | 0.103 | 0.093 | B>A>C>D |
| BM | 0.957 | 0.977 | 0.047 | 0.056 | B>A>D>C |
| CIF | 0.309 | 0.630 | 0.046 | 0.046 | B>A>C=D |
| IP | 0.019 | 0.039 | 0.003 | 0.003 | B>A>C=D |

Table 2: Measures & Ranking of RI Using Parameter Set I

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.530 | 0.060 | A=B>C>D |
| RAW | 16.759 | 16.759 | 8.875 | 1.008 | A=B>C>D |
| RRW | 1.474 | 2.912 | 1.008 | 1.008 | B>A>C=D |
| DIF | 0.335 | 0.670 | 0.009 | 0.504 | B>D>A>C |
| BM | 0.960 | 0.980 | 0.470 | .0009 | B>A>C>D |
| CIF | 0.322 | 0.657 | 0.008 | 0.008 | B>A>C=D |
| IP | 0.019 | 0.039 | .0005 | .0005 | B>A>C=D |

Table 3: Measures & Ranking of RI Using Parameter Set II

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.247 | 0.153 | A=B>C>D |
| RAW | 12.818 | 12.818 | 3.171 | 1.965 | A=B>C>D |
| RRW | 1.318 | 1.970 | 1.318 | 1.318 | B>A=C=D |
| DIF | 0.256 | 0.513 | 0.317 | 0.393 | B>D>C>A |
| BM | 0.941 | 0.960 | 0.188 | 0.094 | B>A>C>D |
| CIF | 0.241 | 0.492 | 0.241 | 0.241 | B>A=C=D |
| IP | 0.019 | 0.038 | 0.019 | 0.019 | B>A=C=D |

Table 4: Measures & Ranking of RI Using Parameter Set III

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.530 | 0.097 | A=B>C>D |
| RAW | 12.818 | 12.818 | 6.788 | 1.241 | A=B>C>D |
| RRW | 1.318 | 1.970 | 1.318 | 1.318 | B>A=C=D |
| DIF | 0.256 | 0.513 | 0.272 | 0.621 | D>B>C>A |
| BM | 0.941 | 0.960 | 0.470 | 0.038 | B>A>C>D |
| CIF | 0.241 | 0.492 | 0.241 | 0.241 | B>A=C=D |
| IP | 0.019 | 0.038 | 0.019 | 0.019 | B>A=C=D |

Table 5: Measures & Ranking of RI Using Parameter Set IV

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.555 | 0.198 | A=B>C>D |
| RAW | 6.513 | 6.513 | 3.611 | 1.290 | A=B>C>D |
| RRW | 1.059 | 2.581 | 1.409 | 1.409 | B>C=D>A |
| DIF | 0.065 | 0.651 | 0.361 | 0.645 | B>D>C>A |
| BM | 0.855 | 0.941 | 0.446 | 0.089 | B>A>C>D |
| CIF | 0.056 | 0.613 | 0.290 | 0.290 | B>C=D>A |
| IP | 0.009 | 0.094 | 0.045 | 0.045 | B>C=D>A |

Table 6: Measures & Ranking of RI Using Parameter Set V

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.53 | 0.116 | A=B>C>D |
| RAW | 11.439 | 11.439 | 6.058 | 1.323 | A=B>C>D |
| RRW | 1.271 | 1.77 | 1.477 | 1.477 | B>C=D>A |
| DIF | 0.229 | 0.458 | 0.364 | 0.661 | D>B>C>A |
| BM | 0.931 | 0.952 | 0.470 | 0.056 | B>A>C>D |
| CIF | 0.213 | 0.435 | 0.323 | 0.323 | B>C=D>A |
| IP | 0.019 | 0.038 | 0.028 | 0.028 | B>C=D>A |

Table 7: Measures & Ranking of RI Using Parameter Set VI

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.53 | 0.135 | A=B>C>D |
| RAW | 10.327 | 10.327 | 5.469 | 1.389 | A=B>C>D |
| RRW | 1.235 | 1.636 | 1.636 | 1.636 | C=D=B>A |
| DIF | 0.207 | 0.413 | 0.438 | 0.694 | D>C>B>A |
| BM | 0.922 | 0.941 | 0.470 | 0.075 | B>A>C>D |
| CIF | 0.190 | 0.389 | 0.389 | 0.389 | C=D=B>A |
| IP | 0.018 | 0.038 | 0.038 | 0.038 | C=D=B>A |

Table 8: Measures & Ranking of RI Using Parameter Set VII

| Comp. | A | B | C | D | Order |
|---|---|---|---|---|---|
| CP | 1.000 | 1.000 | 0.53 | 0.153 | A=B>C>D |
| RAW | 9.413 | 9.413 | 4.985 | 1.443 | A=B>C>D |
| RRW | 1.207 | 1.54 | 1.795 | 1.795 | C=D>B>A |
| DIF | 0.188 | 0.377 | 0.499 | 0.721 | D>C>B>A |
| BM | 0.912 | 0.931 | 0.470 | 0.094 | B>A>C>D |
| CIF | 0.172 | 0.351 | 0.443 | 0.443 | C=D>B>A |
| IP | 0.018 | 0.037 | 0.047 | 0.047 | C=D>B>A |

Table 9: Measures & Ranking of RI Using Parameter Set VIII

4.2 *Discussions*

Our observations in section 4.1 show that conditional probability (CP), risk achievement worth (RAW), and

Birnbaum's measure (BM) may induce misleading conclusions in terms of guiding system maintenance, though some of these measures serve a good indicator for selecting components that are the best candidates for efforts leading to improving system reliability. Risk reduction worth (RRW), criticality importance factor (CIF), and improvement potential (IP) generally induce reasonable conclusions. But they give the same result for all components in a parallel structure irrespective of the (drastic) difference among the component reliabilities. Under this circumstance, the maintainer would lack of a deterministic order to check the failed components in the case of a system failure. In addition, the CIF and IP measures become impractical for large dynamic systems, which must be solved using Markov approaches (Ref. 3), due to the well-known state explosion problem of Markov approaches. Also, the assessment of $I^{BM}(e)$ in both measures involves simultaneously solving a set of differential equations (the number of equations is the same as the number of states present in the Markov model) for the state occupation probabilities and a much larger set of partial differential equations for the component importance analysis (Ref. 5). The solutions to those equations are computationally intensive.

As a result of our current study, we propose the DIF measure to be the most informative and appropriate measure for the maintenance-oriented importance analysis. The DIF measure generally produces the ranking that is consistent with those produced by using the RRW, CIF, and IP measures; it accounts for the effects of exceptionally unreliable component; it can always distinguish components that occupy similar structural positions (for both series and parallel structures) but have different reliabilities.

In the following section, we propose the basic algorithm to compute the DIF measure.

## 5. *ALGORITHM TO COMPUTE DIF*

In section 2.4, we present the definition of the DIF measure as (eq. 4) $I^{DIF}(e) = \Pr\{e \mid S\} = \dfrac{\Pr\{S \cap e\}}{\Pr\{S\}} = \dfrac{\Pr\{S \cap e\}}{U_{sys}}$.

The problem of computing the DIF measure is actually a problem of finding $Pr\{S \cap e\}$ and $U_{sys}$. $U_{sys}$ is the probability of the occurrence of the top event of the system fault tree model. To calculate $Pr\{S \cap e\}$, we generate a new fault tree by applying AND logic operation between the original system fault tree and the basic event *e*. Thus, the top event of the new fault tree is $S \cap e$, and the analysis of the new fault tree gives $Pr\{S \cap e\}$. Figure 2 gives a conceptual overview of the fault tree approach to computing the DIF measure.

In the following of this section, we give a closer look at the approach to solving a fault tree. Solving a fault tree involves using appropriate techniques for finding the probability of occurrence of the top event based on the probability of occurrence of the basic events. Traditional or static fault trees (which have only static gates AND, OR, and K-out-of-N, Ref. 3) can be solved with a variety of techniques such as minimal cut-sets with inclusion-exclusion, minimal cut-sets with sum of disjoint products, and binary decision diagrams (BDD). It has been shown that BDD is the most efficient solution technique for solving the static fault trees. Fault trees that include dynamic gates such as FDEP, PAND, SEQ, and SPARE gates (Ref. 3) cannot be solved with those techniques. Instead, dynamic fault trees are translated into Markov chains for solutions. The biggest drawback of Markov approach is the well-known state explosion problem, which limits the size of the system to be solved. It is for this reason that Ref. 6 proposed a modular approach that can integrate the Markov chain based approach with the BDD solution wherever possible. The modular approach implements the process of finding the independent modules in a fault tree and solving the static subtrees using the BDD technique and the dynamic subtrees using Markov-based technique, and integrating the solutions of those independent subtrees to obtain the probability of occurrence of the top event, that is, the system unreliability.

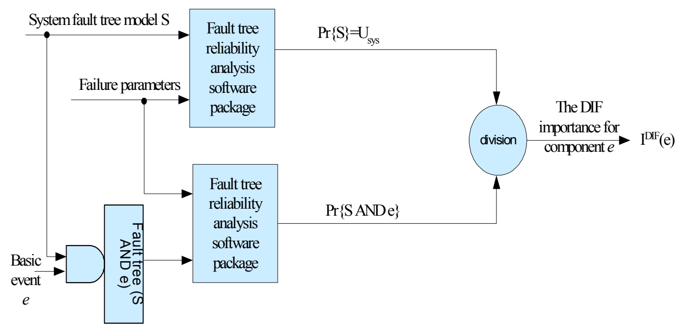


Figure 2: A Conceptual Overview of Approach for Computing DIF Measure

## 6. *INCORPORATION OF COMMON CAUSE FAILURES*

Practical systems can be subject to common-cause failures. Common-cause failures (CCF) are multiple failures that are a direct result of a common cause (CC) or a shared root cause (Ref. 7), such as extreme environmental conditions, human operation and maintenance errors. Examples abound in the real world. Sabotage, earthquake, hurricane, and power outage can obviously cause the simultaneous failure of numerous components in a system. The challenge from CCF is to cope with multiple dependent faults at the same time. In this section, we present an efficient decomposed approach for incorporating CCF into the MO-IA using the DIF measure, based on our discussion in Ref. 9.

First we denote the CC related to a system as $CC_1$, $CC_2$, …, $CC_m$, where *m* is the total number of CC related to the system. The *m* common-causes partition the event space into the following $2^m$ disjoint subsets, each called a common-cause event (CCE):

$$CCE_1 = \overline{CC_1} \cap \overline{CC_2} \cap \ldots \cap \overline{CC_m},$$
$$CCE_2 = CC_{11} \cap \overline{CC_2} \cap \ldots \cap \overline{CC_m},$$
$$\ldots\ldots,$$
$$CCE_{2^m} = CC_{11} \cap CC_2 \cap \ldots \cap CC_m.$$

 

Thus, we can build a space called "CCE space" over this set of collectively exhaustive and mutually exclusive common-cause events that can occur in a system, that is, $\Omega_{CCE} = \{CCE_1, CCE_2, ..., CCE_{2^m}\}$. If $P(CCE_j)$ denotes the probability of $CCE_j$ occurring, then we have $\sum_{j=1}^{2^m} P(CCE_j) = 1$ and $CCE_i \cap CCE_j = \phi$ for any $i \neq j$.

Based on the above CCE space and the "total probability theorem", we can calculate the probability of occurrence of the top event, i.e., the system unreliability, as:

$$U_{sys} = \sum_{i=1}^{2^m} [\Pr(\text{system fails} \mid CCE_i) \bullet P(CCE_i)] \quad (9)$$

where Pr{system fails|$CCE_i$} is a conditional probability that the system fails conditioned on the occurrence of $CCE_i$. It is actually a reduced fault tree reliability problem in which the components affected by $CCE_i$ do not appear. Specifically, in the system fault tree model, each basic event (the failure of a component) that is affected by $CCE_i$ will be replaced by a constant logic value '1' (True). After the replacement, a Boolean reduction can be applied to the system fault tree to generate a simpler fault tree in which all the components affected by $CCE_i$ do not appear. Most importantly, the evaluation of the reduced fault tree can proceed without further consideration of common-cause failures. Thereby, the overall solution complexity is reduced. In summary, the method decomposes a fault tree reliability problem with CCF into a number of reduced reliability problems in which the CCF are factored out. Figure 3 shows a conceptual overview of the decomposed approach for incorporating the CCF.

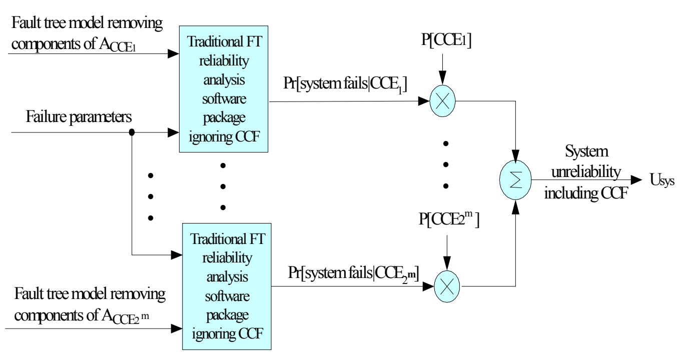


Figure 3: A Conceptual Overview of the Decomposed Approach for Incorporating CCF

Finally, we can incorporate the CCF into the component importance analysis by applying the decomposed approach to the solution of $Pr\{S \cap e\}$ and $U_{sys}$ in the definition of the DIF measure (eq. 4).

## REFERENCES


1. J. Andrews and T. Moss, *Reliability and Risk Assessment*, 1993, Longman Scientific and Technical, Essex.
2. A. Anne, *Implementation of Sensitivity Measures for Static and Dynamic Subtrees in DIFtree,* 1997, M.S. Thesis, University of Virginia.
3. J. B. Dugan and S. A. Doyle, "New results in fault-tree analysis", *Tutorial Notes Ann. Reliability and Maintainability Symp.*, 1996 Jan.
4. Y. Dutuit and A. Rauzy, "Efficient algorithm to assess component and gate importance in fault tree analysis", *Reliability Eng and System Safety*, vol. 72, 2001, pp 213-222.
5. P. M. Frank, *Introduction to System Sensitivity*, 1978, Academic Press.
6. R. Gulati and J. B. Dugan, " A modular approach for analyzing static and dynamic fault trees", *Proc. Ann. Reliability and Maintainability Symp.*, 1997 Jan.
7. A. Hoyland and M. Rausand, *System Reliability Theory: Models and Statistical Methods*, 1994, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons.
8. A. Papoulis, Probability, *Random Variables, and Stochastic Processes (3rd Edition)*, 1991, McGraw-Hill Series in Electrical Engineering, McGraw-Hill.
9. L. Xing, "Reliability analysis of fault-tolerant systems with common-cause failures", *Proc. The Intl. Conf. on Dependable Systems and Networks*, 2003 June.


## BIOGRAPHY


Liudong Xing, Ph.D.
Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth
285 Old Westport Road
North Dartmouth, MA 02747 USA

e-mail: lxing@umassd.edu


Liudong Xing received her B.E. degree in Computer Science from Zhengzhou University, China, in 1996, and was a Research Assistant in Shenyang Institute of Computing Technology, Chinese Academy of Science, China from 1996 to 1998. She was awarded the M.S. and Ph.D. degree in Electrical Engineering from the University of Virginia, Charlottesville, in 2000 and 2002, respectively. Dr. Xing is an Assistant Professor of Electrical and Computer Engineering Department at the University of Massachusetts, Dartmouth. Her major research interests lie in the design and analysis of dependable computer-based systems and networks. She is a member of the IEEE.