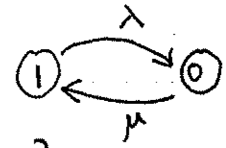
[redacted] Hands-on Problem



1. state equations: $A \cdot p(t) = \dot{p}(t)$

$$\begin{bmatrix} -\alpha_{00} & \alpha_{10} \\ \alpha_{01} & -\alpha_{11} \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \end{bmatrix} = \begin{bmatrix} \dot{p}_0(t) \\ \dot{p}_1(t) \end{bmatrix}$$

$$\begin{bmatrix} -\mu & \lambda \\ \mu & -\lambda \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \end{bmatrix} = \begin{bmatrix} \dot{p}_0(t) \\ \dot{p}_1(t) \end{bmatrix}$$

2. According to Eq.[LTSE], we have

$$\begin{bmatrix} -\mu & \lambda \\ \mu & -\lambda \end{bmatrix} \begin{bmatrix} p_0^*(s) \\ p_1^*(s) \end{bmatrix} = \begin{bmatrix} s\, p_0^*(s) \\ s\, p_1^*(s) \end{bmatrix} - \begin{bmatrix} p_0(0) \\ p_1(0) \end{bmatrix}$$

Initial condition $p_0(0) = 0 \quad p_1(0) = 1$

$$= \begin{bmatrix} s\, p_0^*(s) \\ s\, p_1^*(s) - 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -\mu\, p_0^*(s) + \lambda\, p_1^*(s) = s\, p_0^*(s) & ① \\ \mu\, p_0^*(s) - \lambda\, p_1^*(s) = s\, p_1^*(s) - 1 & ② \end{cases}$$

a set of linear equations

Adding ① & ②:

$$s\, p_0^*(s) + s\, p_1^*(s) - 1 = 0 \Rightarrow p_0^*(s) = \frac{1}{s} - p_1^*(s)$$

Inserting this expression into ②:

$$\frac{\mu}{s} - \mu\, p_1^*(s) - \lambda\, p_1^*(s) = s\, p_1^*(s) - 1 \Rightarrow$$

$$p_1^*(s) = \frac{1 + \frac{\mu}{s}}{s + \lambda + \mu}$$

To find the inverse Laplace transform, we rewrite the expression as:

$$p_1^*(s) = \frac{1}{\lambda+\mu+s} + \frac{\mu}{s} \cdot \frac{1}{\lambda+\mu+s} = \frac{s+\mu}{(\lambda+\mu+s)\cdot s} = \frac{(s+\mu)(\lambda+\mu)}{(\lambda+\mu)(\lambda+\mu+s)\cdot s}$$

$$= \frac{s\lambda + \mu(\lambda+s+\mu)}{(\lambda+\mu)(\lambda+\mu+s)\cdot s} = \frac{\lambda}{\lambda+\mu} \cdot \frac{1}{\lambda+\mu+s} + \frac{\mu}{\lambda+\mu} \cdot \frac{1}{s}$$

According to $e^{\alpha t} \Leftrightarrow \frac{1}{s-\alpha}$, $1 \Leftrightarrow \frac{1}{s}$, we have

$$* \quad p_1(t) = \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\mu}{\lambda+\mu} \qquad [\alpha = -(\lambda+\mu)]$$

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$$p_0(t) = 1 - p_1(t)$$
$$= -\frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\lambda}{\lambda+\mu}$$

As a note:

for reparable systems, we are interested in availability.

system availability $A_{sys}(t) = p_1(t) = \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\mu}{\lambda+\mu}$.