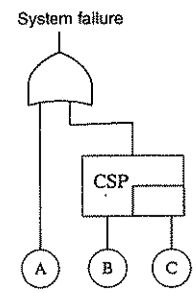
[redacted]

Hands-On Problem

- For the fault tree model, assume components A and B have the failure rate of λ_A and λ_B, respectively. Component C has the failure rate of λ_C after being activated.
  - find the state transition diagram of the Markov chain
  - find the state equations for the time-dependent solution
  - find the state equations of the asymptotic solution
  - find the Laplace transform of the time-dependent state probabilities
  - find the system unreliability in the steady-state



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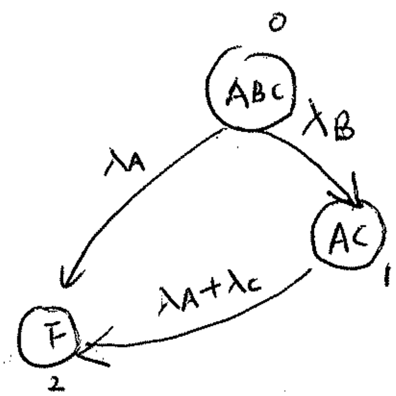
(a)



(b)

$$\begin{bmatrix} -(\lambda_A+\lambda_B) & 0 & 0 \\ \lambda_B & -(\lambda_A+\lambda_C) & 0 \\ \lambda_A & \lambda_A+\lambda_C & 0 \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \end{bmatrix} = \begin{bmatrix} \dot{P}_0(t) \\ \dot{P}_1(t) \\ \dot{P}_2(t) \end{bmatrix}$$

(c)

$$\begin{bmatrix} & & \\ & \text{same as in (b)} & \\ & & \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or So

Balance Equations:

Rate in = Rate out

$$0 = P_0 * (\lambda_A+\lambda_B)$$

$$\lambda_B \cdot P_0 = (\lambda_A+\lambda_C) \cdot P_1$$

$$\lambda_A \cdot P_0 + (\lambda_A+\lambda_C) \cdot P_1 = 0 \cdot P_2$$

(d) Apply Laplace Transform on both sides of (b)

$$\begin{bmatrix} -(\lambda_A+\lambda_B) & 0 & 0 \\ \lambda_B & -(\lambda_A+\lambda_C) & 0 \\ \lambda_A & \lambda_A+\lambda_C & 0 \end{bmatrix} \begin{bmatrix} P_0^*(s) \\ P_1^*(s) \\ P_2^*(s) \end{bmatrix} = \begin{bmatrix} s P_0^*(s) - P_0(0) \\ s\cdot P_1^*(s) - P_1(0) \\ s\cdot P_2^*(s) - P_2(0) \end{bmatrix} = \begin{bmatrix} s\cdot P_0^*(s) - 1 \\ s\cdot P_1^*(s) \\ s\cdot P_2^*(s) \end{bmatrix}$$

Also, $P_0^*(s) + P_1^*(s) + P_2^*(s) = \frac{1}{s}$

$$-(\lambda_A+\lambda_B)\cdot P_0^*(s) = s\cdot P_0^*(s) - 1 \Rightarrow P_0^*(s) = \frac{1}{s+\lambda_A+\lambda_B}$$

$\Rightarrow$

$$\lambda_B\cdot P_0^*(s) - (\lambda_A+\lambda_C)\cdot P_1^*(s) = s\cdot P_1^*(s) \Rightarrow P_1^*(s) = \frac{\lambda_B\cdot P_0^*(s)}{s+\lambda_A+\lambda_C}$$

$$= \frac{\lambda_B}{(s+\lambda_A+\lambda_C)(s+\lambda_A+\lambda_B)}$$

$$P_2^*(s) = \frac{1}{s} - P_0^*(s) - P_1^*(s)$$

$$= \frac{1}{s} - \frac{1}{s+\lambda_A+\lambda_B} - \frac{\lambda_B}{(s+\lambda_A+\lambda_C)(s+\lambda_A+\lambda_B)}$$

(e) Based on (c)

$P_0 = 0$

$P_1 = 0$

$P_2 = 1 - P_0 - P_1 = 1$