Problem 1: a) 10 points b) 5 points, c) 5 points d) 15 points

System failure

(a) ①
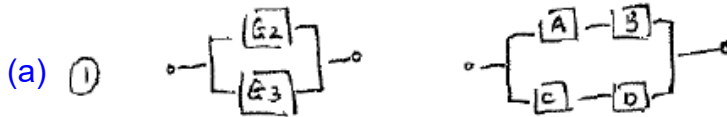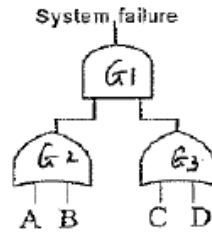
(c) ② Minimal path sets: $P_1 = \{A, B\}$ $P_2 = \{C, D\}$

(b) minimal Cut sets:

G_1

G_2, G_3

$A\,G_3$ $B\,G_3$

AC AD BC BD
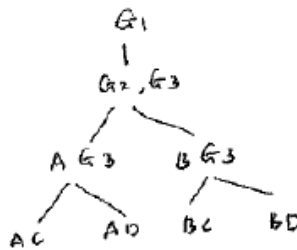
$C_1 = \{A, C\}$ $C_2 = \{A, D\}$

$C_3 = \{B, C\}$ $C_4 = \{B, D\}$

(d) ③

I/E based on path sets:

$R_{sys} = Pr\{P_1 \cup P_2\}$

$= Pr(P_1) + Pr(P_2) - Pr(P_1 \cap P_2)$

$= P(A)\cdot P(B) + P(C)P(D) - P(A)\cdot P(B)\cdot P(C)P(D)$

$= 0.99 \times 0.8 + 0.97 \times 0.9 - 0.99 \times 0.8 \times 0.97 \times 0.9$

$= 0.973584$

SDP based on path sets

$R_{sys} = Pr\{P_1 \cup P_2\}$

$= Pr(P_1) + Pr(\overline{P_1} P_2)$

$= P(A)P(B) + Pr(\overline{AB}CD)$

$= P(A)P(B) + Pr(\overline{A}CD + \overline{B}CD)$

$= P(A)P(B) + Pr(\overline{A}CD) + Pr(\overline{B}CD) -$
    $Pr(\overline{A}\,\overline{B}\,CD)$

$= 0.99 \times 0.8 + 0.10 \times 0.97 \times 0.9 +$
    $0.2 \times 0.97 \times 0.9 - 0.1 \times 0.2 \times 0.97 \times$

$= 0.973584$

a) $RBD \rightarrow FT$

G1

G2

G3

G4

D
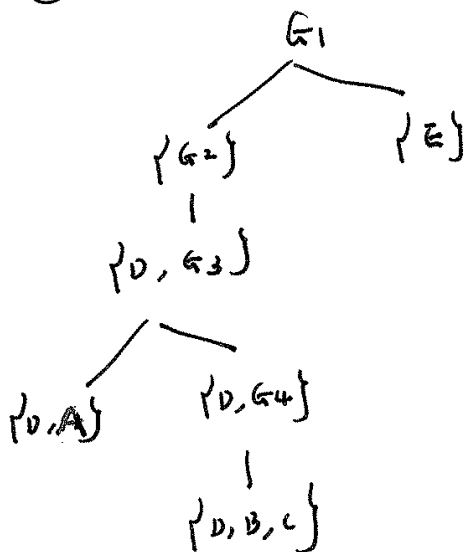
A

E

B   C
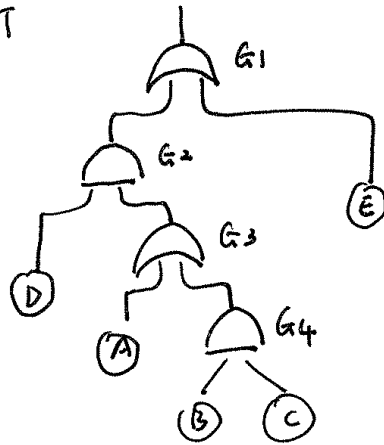
b) Based on RBD model; the minimal path sets are:

$$P_1 = \{A, B, E\} \quad P_2 = \{A, C, E\} \quad P_3 = \{D, E\}$$

c) Based on FT model; the minimal cut sets are:

G1

$\{G2\}$ $\{E\}$

$\{D, G3\}$

$\{D, A\}$ $\{D, G4\}$

$\{D, B, C\}$

$$C_1 = \{E\}$$
$$C_2 = \{A, D\}$$
$$C_3 = \{B, C, D\}$$

d) Solution using the SDP method based on cutsets generated in part c)
   (Alternative methods: cutset using I/E, pathset using SDP, pathset using I/E)

(d) $U_{sys} = Pr\{C_1 \cup C_2 \cup C_3\}$

$\qquad = Pr\{C_1\} + Pr\{\bar{C_1} C_2\} + Pr\{\bar{C_1} \bar{C_2} C_3\}$

$Pr\{C_1\} = Pr\{E\} = 0.1$

$Pr\{C_2\} = Pr\{\bar{E} \cdot AD\} = 0.9 * 0.1 * 0.1 = 0.009$

$Pr\{\bar{C_1} \bar{C_2} C_3\} = Pr\{\bar{E} \, \overline{AD} \, BCD\} = Pr\{\bar{E}(\bar{A}+\bar{D}) \cdot BCD\}$

$\qquad = Pr\{\bar{A} \bar{E} BCD\} = 0.9^2 * 0.1^3 = 0.0008$

$U_{sys} = 0.1 + 0.009 + 0.0008$

$\qquad = 0.1098$

$R_{sys} = 1 - U_{sys} = 0.8901$

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e) Solution using the SDP method based on cutsets generated in part c)
(Alternative methods: cutset using I/E, pathset using SDP, pathset using I/E)

(e) Similar to (d)

$U_{sys} = Pr\{C_1\} + Pr\{\bar{C_1} C_2\} + Pr\{\bar{C_1} \bar{C_2} C_3\}$

$Pr\{C_1\} = Pr\{E\} = 1 - e^{-\lambda t} = 1 - e^{-0.1 * 10} = 0.6321 2|$

$Pr\{\bar{C_1} C_2\} = Pr\{\bar{E} AD\} = (1 - 0.6321) * 0.632|^2 = 0.146996$

$Pr\{\bar{C_1} \bar{C_2} C_3\} = Pr\{\bar{A} \bar{E} BCD\} = (1 - 0.6321)^2 * 0.6321^3 = 0.034183$

$U_{sys} = 0.63212| + 0.146996 + 0.034183 = 0.8133$

$R_{sys} = 1 - U_{sys} = 0.1867$