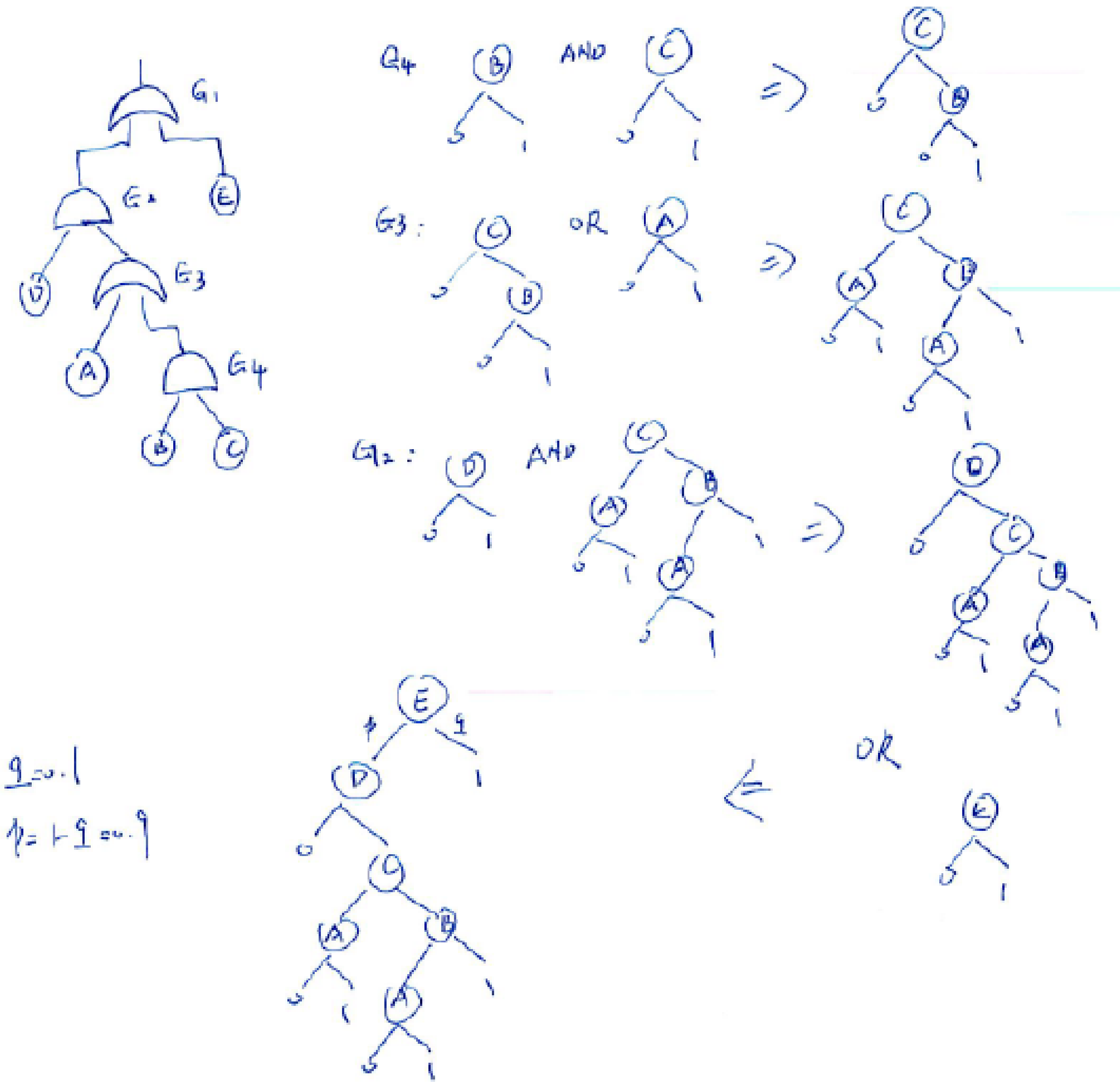
ECE454/544: Fault-Tolerant Computing & Reliability Engineering (Fall 2022)

Homework #6 Solution

(100 points)

1. (40 points) Consider the following fault tree model for a system with five components A, B, C, D, and E. Gates G1 and G3 are OR gates; gates G2 and G4 are AND gates.
    a. (20 points) Generate the binary decision diagram (BDD) for the fault tree using ordering E<D<C<B<A.



b. (10 points) Assume the **failure probability** for each component is 0.1. Find the system reliability at time t=10 hours.

p=0.9
q=0.1
$R_{sys}=p_E p_D + p_E q_D p_C p_A + p_E q_D q_C p_B p_A$
$=p^2 + p^3 q + p^3 q^2$
$= 0.9^2 + 0.9^3 * 0.1 + 0.9^3 * 0.1^2$
= 0.89019

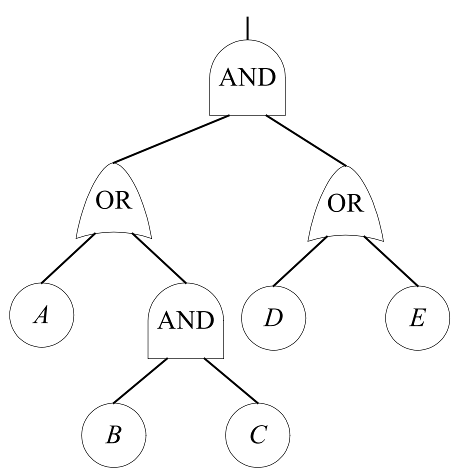
c. (10 points) Assume the **failure rate** for each component is 0.1/hour. Find the system reliability at time t=10 hours.

Each component's time to failure follows the exponential distribution. Thus, the component reliability and unreliability can be evaluated as (4 points):
p=exp(-0.1*10)=0.367879
q=1-p=0.632121

(6 points) Using the same reliability expression as in b)
$R_{sys}= p_E p_D + p_E q_D p_C p_A + p_E q_D q_C p_B p_A$
$=p^2 + p^3 q + p^3 q^2$
$= 0.367879^2 + 0.367879^3 * 0.632121 + 0.367879^3 * 0.632121^2$
= 0.1867

2. (60 points) Consider the following system fault tree model. Assume the failure probability for each component is:

Component	A	B	C	D	E
Failure probability	0.2	0.2	0.1	0.3	0.3

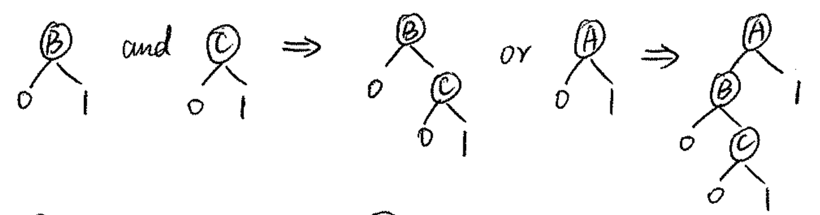


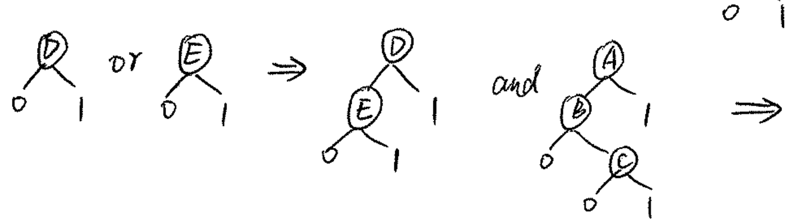
a. (20 points) Find the system reliability at time t=1000 hours using **the BDD method**.

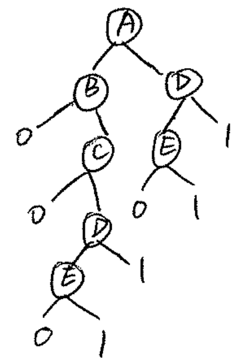
b. (25 points) Rank the importance of the five components using the Birnbaum's measure

c. (15 points) Find the importance value of component *B* using the diagnostic importance factor (DIF)

a)







$$R_{sys} = q_A \cdot (1-q_D)(1-q_E) + (1-q_A)\cdot(1-q_B) + (1-q_A)\cdot q_B \cdot (1-q_C) + (1-q_A)\cdot q_B \cdot q_C \cdot (1-q_D)\cdot(1-q_E)$$

$$= 0.2\times(1-0.3)\times(1-0.3) + (1-0.2)\times(1-0.2) + (1-0.2)\times 0.2\times(1-0.1) + (1-0.2)\times 0.2\times 0.1\times(1-0.3)\times(1-0.3)$$

$$= 0.098 + 0.64 + 0.144 + 0.00784 = 0.88984$$

b) $U_{sys} = q_A q_D + q_A \cdot (1-q_D) \cdot q_E + (1-q_A) \cdot q_B \cdot q_C \cdot q_D + (1-q_A) \cdot q_B \cdot q_C \cdot (1-q_D) \cdot q_E$

$= q_A q_D + q_A q_E - q_A q_D q_E + q_B q_C q_D - q_A q_B q_C q_D + q_B q_C q_E - q_A q_B q_C q_E$
$- q_B q_C q_D q_E + q_A q_B q_C q_D q_E$

$= 1 - R_{sys} = 1 - 0.88984 = 0.11016$

$$I^{BM}(A) = \frac{\partial sys}{\partial q_A} = q_D + q_E - q_D q_E - q_B q_C q_D - q_B q_C q_E + q_B q_C q_D q_E$$

$= 0.3 + 0.3 - 0.3^2 - 0.2 \times 0.1 \times 0.3 - 0.2 \times 0.1 \times 0.3 + 0.2 \times 0.1 \times 0.3^2$

$= 0.6 - 0.09 - 0.012 + 0.0018 = 0.4998$

$$I^{BM}(B) = \frac{\partial sys}{\partial q_B} = q_C q_D - q_A q_C q_D + q_C q_E - q_A q_C q_E - q_C q_D q_E + q_A q_C q_D q_E$$

$= 0.1 \times 0.3 - 0.2 \times 0.1 \times 0.3 + 0.1 \times 0.3 - 0.2 \times 0.1 \times 0.3 - 0.1 \times 0.3^2 + 0.2 \times 0.1 \times 0.3^2$

$= 0.06 - 0.012 - 0.009 + 0.0018 = 0.0408$

$$I^{BM}(C) = \frac{\partial sys}{\partial q_C} = q_B q_D - q_A q_B q_D + q_B q_E - q_A q_B q_E - q_B q_D q_E + q_A q_B q_D q_E$$

$= 0.2 \times 0.3 - 0.2^2 \times 0.3 + 0.2 \times 0.3 - 0.2^2 \times 0.3 - 0.2 \times 0.3^2 + 0.2^2 \times 0.3^2$

$= 0.12 - 0.024 - 0.018 + 0.0036 = 0.0816$

$$I^{BM}(D) = \frac{\partial sys}{q_D} = q_A - q_A q_E + q_B q_C - q_A q_B q_C - q_B q_C q_E + q_A q_B q_C q_E$$

$= 0.2 - 0.2 \times 0.3 + 0.2 \times 0.1 - 0.2^2 \times 0.1 - 0.2 \times 0.1 \times 0.3 + 0.2^2 \times 0.1 \times 0.3$

$= 0.2 - 0.06 + 0.02 - 0.004 - 0.006 + 0.0012 = 0.1512$

$$I^{BM}(E) = \frac{\partial sys}{q_E} = q_A - q_A q_D + q_B q_C - q_A q_B q_C - q_B q_C q_D + q_A q_B q_C q_D$$

$= 0.2 - 0.2 \times 0.3 + 0.2 \times 0.1 - 0.2^2 \times 0.1 - 0.2 \times 0.1 \times 0.3 + 0.2^2 \times 0.1 \times 0.3$

$= 0.2 - 0.06 + 0.02 - 0.004 - 0.006 + 0.0012 = 0.1512$

Using Birnbaum's measure : $A > D = E > C > B$

c) $I^{DIF}(B) = Pr\{B|S\} = \frac{Pr\{B \cap S\}}{Pr\{S\}}$

$S = (A+BC)\cdot(D+E) \qquad B \cap S = B(A+BC)\cdot(D+E) = (AB+BC)\cdot(D+E)$

~~$= ABD + ABE + BCD + BCE$~~

$Pr\{AB+BC\} = Pr\{AB\} + Pr\{BC\} - Pr\{ABC\} = q_A q_B + q_B q_C - q_A q_B q_C$

$= 0.2^2 + 0.2 \times 0.1 - 0.2^2 \times 0.1 = 0.04 + 0.02 - 0.004 = 0.056$

$Pr\{D+E\} = Pr\{D\} + Pr\{E\} - Pr\{DE\} = q_D + q_E - q_D \cdot q_E$

$= 0.3 + 0.3 - 0.3^2 = 0.6 - 0.09 = 0.51$

$Pr\{B \cap S\} = Pr\{AB+BC\} \cdot Pr\{D+E\} = 0.056 \times 0.51 = 0.02856$

$I^{DIF}(B) = \frac{Pr\{B \cap S\}}{Pr\{S\}} = \frac{0.02856}{U_{sys}} = \frac{0.02856}{0.11016} \approx 0.2593$