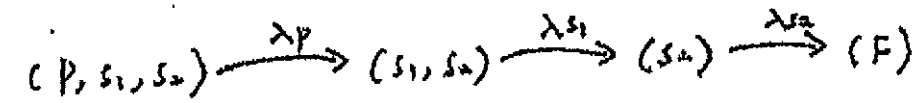
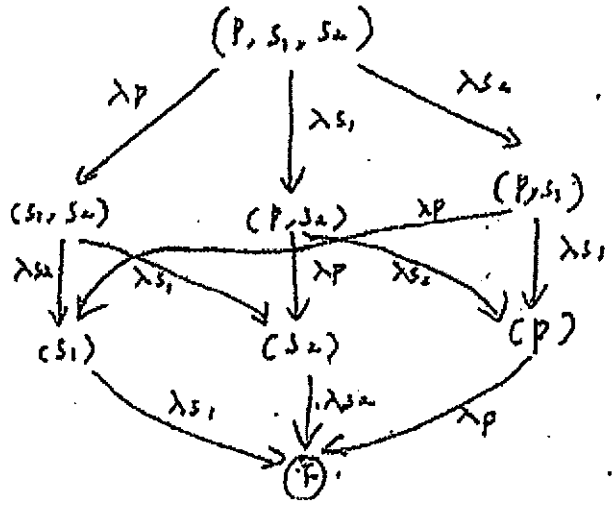
**ECE454/544: Fault-Tolerant Computing & Reliability Engineering**
**Homework #7 Solution**
(65 points)

1. (30 points) Problem 1 solution:

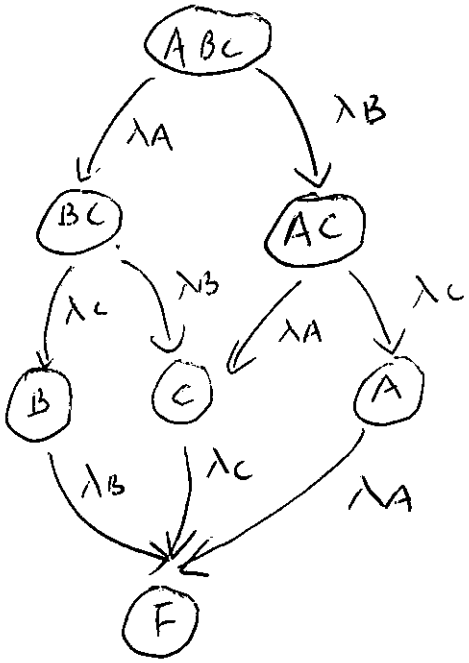
(a)



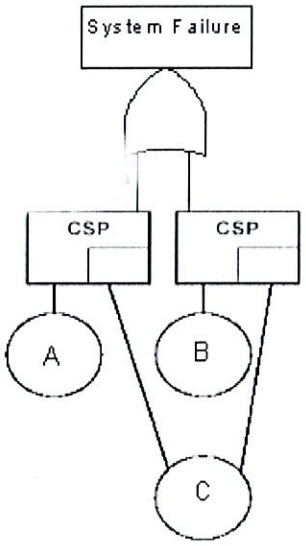

(b)



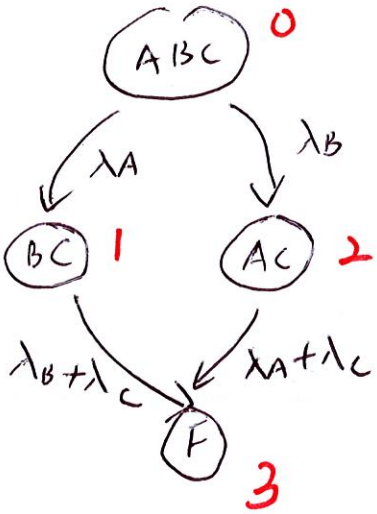

(c)



2. (35 points) For the fault tree model below, assume components A and B have the failure rate of $\lambda_A$ and $\lambda_B$, respectively. Component C has the failure rate of $\lambda_C$ after being activated to replace a failed primary component.
   a) (10 points) find the state transition diagram of the Markov chain
   b) (8 points) find the state equations for the **time-dependent** solution
   c) (7 points) find the state equations of the **asymptotic** solution
   d) (10 points) find the system unreliability in the **steady-state** by solving the state equations of b)




a)




b)

$$\begin{bmatrix} -(\lambda_A+\lambda_B) & 0 & 0 & 0 \\ \lambda_A & -(\lambda_B+\lambda_C) & 0 & 0 \\ \lambda_B & 0 & -(\lambda_A+\lambda_C) & 0 \\ 0 & \lambda_B+\lambda_C & \lambda_A+\lambda_C & 0 \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} = \begin{bmatrix} \dot{P}_0(t) \\ \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \end{bmatrix}$$

c)

$$\begin{bmatrix} -(\lambda_A+\lambda_B) & 0 & 0 & 0 \\ \lambda_A & -(\lambda_B+\lambda_C) & 0 & 0 \\ \lambda_B & 0 & -(\lambda_A+\lambda_C) & 0 \\ 0 & \lambda_B+\lambda_C & \lambda_A+\lambda_C & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

d)

$$\begin{cases} -(\lambda_A+\lambda_B)P_0 + 0 = 0 & ① \\ \lambda_A P_0 - (\lambda_B+\lambda_C)P_1 + 0 = 0 & ② \\ \lambda_B P_0 - (\lambda_A+\lambda_C)P_2 = 0 & ③ \\ P_0+P_1+P_2+P_3 = 1 & ④ \end{cases}$$

By ①: $P_0 = 0$

By ②: $P_1 = 0$

By ③ $P_2 = 0$

By ④: $P_3 = 1 - P_0 - P_1 - P_2 = 1$