ECE 454/544: Fault-Tolerant Computing & Reliability Engineering

Lecture #17 – **Network Reliability**

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Fall 2022

Administrative Issues

(Nov. 21, Monday)

- Homework#7 (last one☺)
  - Due **Today**
- Project final report
  - Due by **Nov. 30, Wednesday**
  - Please check out the Report Guidelines for requirements
- Project presentation slides
  - Due by **Dec. 5, Monday**
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Topics

- Graph models and definitions
- Evaluation approaches

Network Graph Model

- A probabilistic graph G=(V, E)
  - V: a set of nodes/vertices (network elements, such as terminals and routers)
  - E: a set of links (communication paths)
  - with failure probabilities assigned to each node and/or link in the graph.

Definitions of Network Reliability

- In general, the probability that the system operates successfully for a given period time under environmental conditions (temperature, atmosphere, weather, system load and traffic)

- Reliability of a network G
  - probability that G supports a given operation

Definition (Cont'd)

- Three kinds of network reliability
  - Two-terminal reliability (terminal-pair): two given vertices can communicate
  - All-terminal reliability: operation requires each pair of vertices is able to communicate via at least one operational path
  - K-terminal reliability: operation requires a few vertices, a subset K of V, to communicate each other

Example Evaluation Methods (Two-Terminal Reliability)

- State space enumeration method
- Cut-set ad tie-set method
- Graph transformation method
- Binary decision diagrams-based method

Reference:

1. Xinyu Zang, Hairong Sun, Kishor S. Trivedi, "A BDD-Based Algorithm for Reliability Graph Analysis", http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.44.6169
2. Corinne Lucet, Jean-Francois Manouvrier, "Exact Methods to compute Network Reliability", http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.27.7190
3. Martin L. Shooman, "Reliability of Computer Systems and Networks: Fault Tolerance, Analysis, and Design", John & Sons Wiley, 2002, ISBN 0-471-29342-3 (Chapter 6).

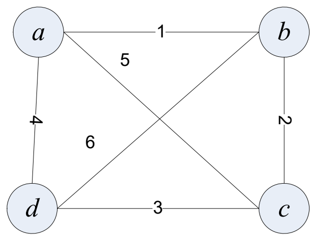
State-Space Enumeration

- Enumerate all possible combinations where each of the edges in E can be good or bad → $2^{|E|}$ disjoint combinations
- Each combination → an event
- Network reliability is the probability of the union of the subset of the events that contain a path between *s* and *t*.

$$R_{NW} = P(E_1 \cup E_2 \cup \ldots) = P(E_1) + P(E_2) + \ldots$$

Hands-On Problem (1)

- Assume all nodes are perfect, each link has a failure probability of $q=0.1$. Find the two-terminal reliability from *a* to *b*?



State-Space Enumeration (Cont'd)

- Procedure is clear

- Time-consuming
- Exponential event space

Cut-set and Tie-set Method

- A tie/path set
  - a group of edges which forms a connection between input and output when traversed in the arrow direction
- A cut set
  - A set of edges which interrupts all connections between input and output when removed from the graph
- Minimal
  - No node or edge is traversed more than once
  - A minimal tie (cut) set has no subset of edges that are a tie (cut) set.

Cut-set and Tie-set Method (Cont'd)

- There are *i* tie sets between *s* and *t*

$$R_{NW} = P(T_1 \cup T_2 \cup ... \cup T_i)$$

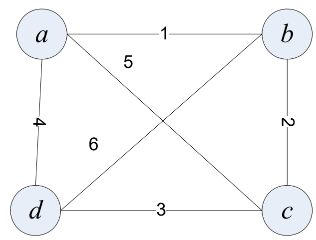
- There are *j* cut sets between *s* and *t*

$$R_{NW} = 1 - P(C_1 \cup C_2 \cup ... \cup C_j)$$

- Solved using I/E or SDP method

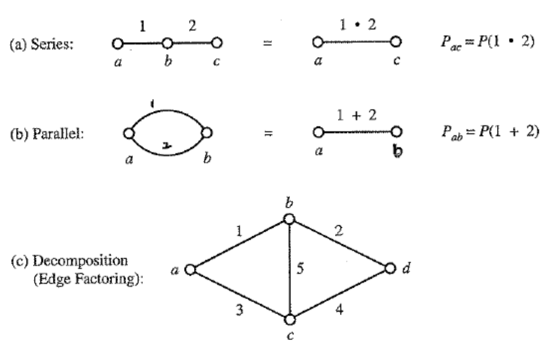
Hands on Problem (2)

- Final the minimal tie sets and cut sets
- Find the two-terminal reliability from *a* to *b*, given that each edge has a failure probability q=0.1.



Transformation Method

- Transform a network into a simpler network (or set of networks) by successively applying transformation
- Three basic transformations



Expand about 5: $R_{ad} = [P(5)Pr(G_1) + P(5')Pr(G_2)]$

Transformation Method (Cont'd)

- Factoring Theorem: For any network reliability measure *R*, probabilistic undirected graph *G*, and edge *e* of *G*, we have:

$$R(G) = p_e R(G \bullet e) + (1 - p_e) R(G - e)$$

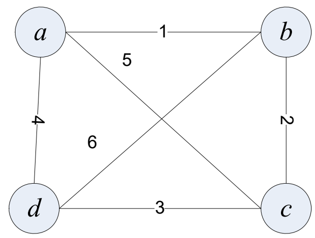
- *G·e* (*G* contract *e*): given e={v1, v2}, *G·e* can be obtained by first removing *e* and then collapsing *v1* and *v2* into a single node *v*; every edge involving *v1* or *v2* is replaced by an edge involving *v*.
- *G-e* (*G* delete *e*): remove the edge *e* from the graph

Transformation Method (Cont'd)

- Use series and parallel transformations first
- Resort to edge-factoring only when no more series or parallel transformations can be made!

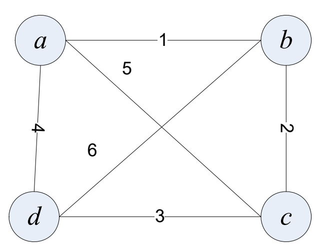
Hands on Problem (3)

- Find the two-terminal reliability from *a to b* using transformation method, given that each edge has a failure probability q=0.1.



Hands on Problem (4)

- Find the two-terminal reliability from *a to c* using transformation method, given that each edge has a failure probability q=0.1.



Example Evaluation Methods (Agenda)

- ✓ State space enumeration method
- ✓ Cut-set ad tie-set method
- ✓ Graph Transformation method
- **Binary decision diagrams based method**

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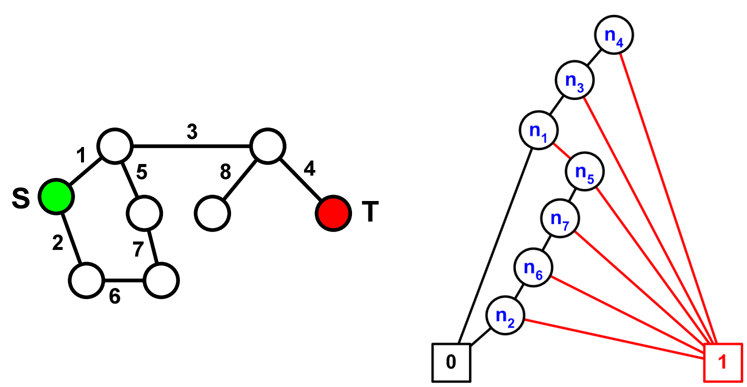
Binary Decision Diagrams Based Method

- Convert the graph into a BDD and then evaluate the BDD



Algorithm for generating BDD from probabilistic graph

- Traverse the graph breadth-first to list all path-sets.
- Convert each pathset to an equivalent BDD by ANDing all the components in that path-set
- Generate total BDD by ORing all path-set BDDs

```
bdd_gen(start_node) {
    T_bdd = 0
    set start_node in this_path
    for (edge_i in the set of edges starting from start_node) {
        next_node = the other end of edge_i
        if (next_node == sink_node)
            subpath_bdd = edge_i_bdd
        else if (next_node is already in this_path)
            continue;
        else
            subpath_bdd = bdd_gen(next_node) AND edge_i_bdd
        T_bdd = T_bdd OR subpath_bdd
    }
    clear start_node in this_path
    return T_bdd
}
```

algorithm proposed by Zang et al.

All-Terminal and K-Terminal Reliability

- State space enumeration method
- Cut-set ad tie-set method
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2. Sy-Yen Kuo, Fu-Min Yeh, and Hung-Yau Lin, "Efficient and Exact Reliability Evaluation for Networks With Imperfect Vertices", IEEE Transactions on Reliability, Vol. 56, No. 2, June 2007. http://ieeexplore.ieee.org/iel5/24/4220773/04220790.pdf
3. Gary Hardy, Corinne Lucet, and Nikolaos Limnios, "K-Terminal Network Reliability Measures With Binary Decision Diagrams", IEEE Transactions on Reliability, Vol. 56, No. 3, September 2007. http://ieeexplore.ieee.org/iel5/24/4298224/04298237.pdf?arnumber=4298237

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